hep-ph/0012208
OCHA-PP-166
HUPD-0011

Revisiting Charmless Two-Body B Decays involving η' and η

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Abstract

We have studied charmless two-body B decays involving η and η' in the framework of QCD improved factorization appraoch. The spectator hard scattering mechanism for η' production have been re-examined and extended, which contributions are incorporated consistently into the factorizable leading contributions. It is found that the conventional mechanism would give $Br(B \to \eta' K) \sim 30 \times 10^{-6}$ which agrees with predictions based on naive factorization approaches. With the incorporation of spectator hard scattering mechanism contributions, theoretical predictions are improved much and the bulk of $Br(B \to \eta' K)$ are accommodated in the reasonable parameter space. We have also presented calculations of $g^* g^* - \eta^{(\prime)}$ transition form factors within the standard hard scattering approach. It is shown that the new contributions in the modes such as $B \to \eta' + vector$ and $B \to \eta + \pi, \rho, K^{(*)}$ are small. Direct CP-violation in those decay modes are predicted. It is shown that the prospects for measuring direct CP-violation effects in $B^{\pm} \to \eta' K^{\pm}$, $\eta' \pi^{\pm}$, $\eta \pi^{\pm}$, and $\eta K^{\pm}$ are promising at B factories in the near future.

PACS Numbers: 13.25Hw,12.15Hh, 12.38Bx

1 Introduction

The first evidence of strong penguin was observed by CLEO[1] in 1997 with the announcement of

$$Br(B^- \to \eta' K^-) = (6.5^{+1.5}_{-1.4} \pm 0.9) \times 10^{-5},$$
$$Br(B^0 \to \eta' K^0) = (4.7^{+2.7}_{-2.0} \pm 0.9) \times 10^{-5}, \tag{1}$$

which are $2 \sim 4$ times larger than any theoretical predictions existed at that time. Driven by the unexpected large data, these decays modes have been investigated extensively [2, 3, 4, 5]. As a result of the investigations in past years, the contribution of the conventional mechanism estimated by using the naive factorization and effective Hamiltonian for B decays would account for 1/4$\sim$ 1/2 of the data. Some new mechanisms are proposed to explain the unexpected large rates of B decays to $K\eta'$. Namely

A)large intrinsic charm content of η'[2] through the chain $b \to c\bar{c}s \to \eta' s$ or through $b \to c\bar{c}s \to sg^*g^* \to \eta' s$[3];

B) strong penguin $b \to sg$[4] enhanced by new physics;

C) spectator hard-scattering mechanism through $g^\star g^\star \to \eta'$[5, 6].

For type A mechanism, its magnitude is characterized by the parameter $f^c_{\eta'}$ defined by the matrix $< \eta' | \bar{c}\gamma_\mu\gamma_5 c | 0 >= -i f^c_{\eta'} p_\mu$. To account for the large branching ratio of $B \to \eta' K$, $f^c_{\eta'}$ would be as large as $(50 \sim 180) MeV$[2]. However, the later analyses have shown $f^c_{\eta'}$ as small as few MeV [7, 8, 9]. It is also realized that strength of the process $b \to c\bar{c}s \to sg^*g^* \to \eta' s$ is very small[10]. Generally compared with uncertainties in form factors and light quark masses in the estimations of nonleptonic B decays, the contribution of type A mechanism for η' exclusive production is probably safe to be neglected. For type B mechanism, it would be very interesting to find signals of new physics beyond the standard model(SM) in these decays if the SM is indeed incapable of accommodating experimental data. In this paper, we study those processes in the SM and investigate the possibility whether the SM can accommodate the present experimental data with new approach for the hadronic dynamics of B decays.

For type C mechanism, it may be promising. Unfortunately it depends on some unknown quantities: the transition form factor of $\eta' - g^*g^*$ and the light cone distribution amplitudes(DA) of the mesons in the process. The prediction in Ref.[5] should be improved and incorporated

with the predictions of the basic mechanisms consistently.

While the data of these decays reported in 1997[1] is still puzzlingly large for theorists, robust experimental investigations are in progress. Recently, using the full CLEO II/II.V data, CLEO Collaborations[11] have improved their previous measurements of $B \to \eta' K$

$$\begin{aligned} \mathcal{B}(B^+ \to \eta' K^+) &= (80^{+10}_{-9} \pm 7) \times 10^{-6}, \\ \mathcal{B}(B^0 \to \eta' K^0) &= (89^{+18}_{-16} \pm 9) \times 10^{-6}, \end{aligned} \tag{2}$$

with observations of two new decay modes

$$\begin{aligned} \mathcal{B}(B^+ \to \eta K^{*+}) &= (26.4^{+9.6}_{-8.2} \pm 3.3) \times 10^{-6}, \\ \mathcal{B}(B^0 \to \eta K^{*0}) &= (13^{+5.5}_{-4.6} \pm 1.6) \times 10^{-6}, \end{aligned} \tag{3}$$

and upper limits for other 12 decay modes involving η or η'.

Theoretically, the importance and generality of the pioneer works of Polizer and Wise[12] and factorization arguments of Bjorken[13] are renewed by Beneke, Buchalla, Neubert and Sachrajda with the formation of "QCD factorization" for B hadronic decays[14, 15]. The factorization formula incorporates elements of the naive factorization approach (as the leading contribution) and the hard-scattering approach(as subleading corrections), which allows us to calculate systematically radiative (subleading nonfactorizable) corrections to naive factorization for B exclusive nonleptonic decays. An important product of the formula is that the strong final-state interaction phases are calculable from the first principle which arise from the hard-scattering kernel and hence process dependent. The strong phases are very important for studying CP violation in B decays. Detailed proofs and arguments could be found in[15]. Here we recall briefly the essence of the QCD factorization formula as follows.

The amplitude of B decays to two light mesons, say M_1 and M_2, is obtained through the hadronic matrix element $\langle M_1(p_1)M_2(p_2)|\mathcal{O}_i|B(p)\rangle$, here M_1 denotes the final meson that picks up the light spectator quark in the B meson, and M_2 is the other meson which is composed of the quarks produced from the weak decay point of b quark. Since the quark pair, forming M_2, is ejected from the decay point of b quark carrying the large energy of order of m_b, soft gluons with the momentum of order of Λ_{QCD} decouple from it at leading order of Λ_{QCD}/m_b in the heavy quark limit. As a consequence any interaction between the quarks of M_2 and the

quarks out of M_2 is hard at leading power in the heavy quark expansion. On the other hand, the light spectator quark carries the momentum of the order of Λ_{QCD}, and is softly transferred into M_1 unless it undergoes a hard interaction. Any soft interaction between the spectator quark and other constituents in B and M_1 can be absorbed into the transition formfactor of $B \to M_1$ which could be extracted from semileptonic decays $B \to M_1 l\bar{\nu}$. The non-factorizable contribution to $B \to M_1 M_2$ can be calculated through the diagrams in Fig.1, which turns out to be subleading oder corrections to factorizable amplitudes.

In this paper we study $B \to \eta^{(\prime)} M (M = \pi, K^{(*)}, \rho)$ decays within the framework of QCD factorization approach [14, 15]. We compare our numerical results with the experimental data presented by CLEO collaboration [11]. We find that the conventional mechanism contributions to $B^+ \to \eta' K^+$ and $B^0 \to \eta' K^0$ are about 27×10^{-6} and 28×10^{-6} respectively, which agree with many theoretical expectations based on naive factorizations. To explain the experimental data, contributions from new mechanisms with the strength as large as $40\% \sim 50\%$ of the strength of the conventional mechanisms are found. Incorporating the contribution of spectator hard-scattering mechanism(SHSM) to these decays, the experimental data could be well accommodated in the SM. SHSM is found to be important for $B \to \eta' K$ but *not* for $B \to \eta' K^*$.

Our predictions agree with the data of measured branching ratios or lie below their upper limits of other decay modes. We also give our predictions of direct CP asymmetries in these decay modes. Direct CP violations in the four observed B decay modes $B^+ \to K^{+*}\eta$ and $B^0 \to K^0\eta'(K^{0*}\eta)$ are found to be about a few percentages, but it can reach 10% for $B^\pm \to K^\pm \eta'$. Considering its large branching ratio, we may expect that direct CP violation effects in charmless B decays would be firstly observed in $B^\pm \to K^\pm \eta'$. Large direct CP violation asymmetries are predicted for the decay modes $B^\pm \to \eta' \pi^\pm$, $\eta\pi^\pm$ and $B^+ \to \eta K^+$. Prospects for observing direct CP-violations in the these decay modes at B factories are very promising.

This paper is organized as following. In Sec.2 we present notations and calculations of the conventional mechanisms contributions to these decays. In Sec.3 we calculate $g^* g^* {-} \eta'$ transition form factors using the standard hard scattering framework of Brodsky and Lepage[16]. With the form factor, we re-examine the contribution of the spectator hard scattering mechanism for $B \to \eta' K$[5] and generalize it to other 14 decay modes. Section 4 contains our numerical results for the branching ratios of two body charmless B decays involving η and η'. Direct CP-violations

in these decays are presented in Sec. 5. Sec. 6 is the summary of our investigations.

2 Calculations of the conventional mechanisms

The contribution of the conventional mechanisms are governed by the effective Hamiltonian for B decays which is given by [17],

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}}\left[V_{ub}V_{uq}^*\left(\sum_{i=1}^{2} C_i O_i^u + \sum_{i=3}^{10} C_i\, O_i + C_g O_g\right) + V_{cb}V_{cq}^*\left(\sum_{i=1}^{2} C_i O_i^c + \sum_{i=3}^{10} C_i O_i + C_g O_g\right)\right], \tag{4}$$

with the effective operators given by

$$\begin{aligned}
O_1^u &= (\bar{q}_\alpha u_\alpha)_{V-A}\cdot(\bar{u}_\beta b_\beta)_{V-A}, & O_2^u &= (\bar{q}_\alpha u_\beta))_{V-A}\cdot(\bar{u}_\beta b_\alpha))_{V-A},\\
O_1^c &= (\bar{q}_\alpha c_\alpha)_{V-A}\cdot(\bar{c}_\beta b_\beta))_{V-A}, & O_2^c &= (\bar{q}_\alpha c_\beta)_{V-A}\cdot(\bar{c}_\beta b_\alpha)_{V-A},\\
O_3 &= (\bar{q}_\alpha b_\alpha)_{V-A}\cdot\textstyle\sum_{q'}(\bar{q}'_\beta q'_\beta)_{V-A}, & O_4 &= (\bar{q}_\alpha b_\beta))_{V-A}\cdot\textstyle\sum_{q'}(\bar{q}'_\beta q'_\alpha)_{V-A},\\
O_5 &= (\bar{q}_\alpha b_\alpha)_{V-A}\cdot\textstyle\sum_{q'}(\bar{q}'_\beta q'_\beta)_{V+A}, & O_6 &= (\bar{q}_\alpha b_\beta)_{V-A}\cdot\textstyle\sum_{q'}(\bar{q}'_\beta q'_\alpha)_{V+A},\\
O_7 &= \tfrac{3}{2}(\bar{q}_\alpha b_\alpha)_{V-A}\cdot\textstyle\sum_{q'} e_{q'}(\bar{q}'_\beta q'_\beta)_{V+A}, & O_8 &= \tfrac{3}{2}(\bar{q}_\alpha b_\beta)_{V-A}\cdot\textstyle\sum_{q'} e_{q'}(\bar{q}'_\beta q'_\alpha)_{V+A},\\
O_9 &= \tfrac{3}{2}(\bar{q}_\alpha b_\alpha)_{V-A}\cdot\textstyle\sum_{q'} e_{q'}(\bar{q}'_\beta q'_\beta)_{V-A}, & O_{10} &= \tfrac{3}{2}(\bar{q}_\alpha b_\beta)_{V-A}\cdot\textstyle\sum_{q'} e_{q'}(\bar{q}'_\beta q'_\alpha\ )_{V-A},\\
O_g &= (g_s/8\pi^2)\, m_b\, \bar{q}_\alpha\, \sigma^{\mu\nu}\, R\,(\lambda^A_{\alpha\beta}/2)\, b_\beta\, G^A_{\mu\nu}\ .
\end{aligned} \tag{5}$$

Here $q = d, s$ and $q'\epsilon\{u, d, s, c, b\}$, α and β are the $SU(3)$ color indices and $\lambda^A_{\alpha\beta}$, $A = 1, ..., 8$ are the Gell-Mann matrices, and $G^A_{\mu\nu}$ denotes the gluonic field strength tensor. The Wilson coefficients evaluated at $\mu = m_b$ scale are[17]

$$\begin{aligned}
C_1 &= 1.082, & C_2 &= -0.185,\\
C_3 &= 0.014, & C_4 &= -0.035,\\
C_5 &= 0.009, & C_6 &= -0.041,\\
C_7 &= -0.002/137, & C_8 &= 0.054/137,\\
C_9 &= -1.292/137, & C_{10} &= 0.262/137,\\
C_g &= -0.143.
\end{aligned} \tag{6}$$

The non-factorizable contributions to $B \to M_1M_2$ can be calculated through the diagrams in Fig.1. The results of our calculations are summarized compactly by the following equations

$$\mathcal{T}_p \;=\; \frac{G_F}{\sqrt{2}}\sum_{p=u,c} V_{pq}^* V_{pb}\Big\{a_1^p(BM_1, M_2)(\bar{q}u)_{V-A}\otimes(\bar{u}b)_{V-A} + a_2^p(BM_1, M_2)(\bar{u}u)_{V-A}\otimes(\bar{q}b)_{V-A}$$

$$
\begin{aligned}
&+ \quad a_3^p(BM_1,M_2)(\bar{q}'q')_{V-A}\otimes(\bar{q}b)_{V-A} + a_4^p(BM_1,M_2)(\bar{q}q')_{V-A}\otimes(\bar{q}'b)_{V-A} \\
&+ \quad a_5^p(BM_1,M_2)(\bar{q}'q')_{V+A}\otimes(\bar{q}b)_{V-A} + a_6^p(BM_1,M_2)(-2)(\bar{q}q')_{S+P}\otimes(\bar{q}'b)_{S-P} \\
&+ \quad a_7^p(BM_1,M_2)\frac{3}{2}e_{q'}(\bar{q}'q')_{V+A}\otimes(\bar{q}b)_{V-A} \\
&+ \quad (-2)\left(a_8^p(BM_1,M_2)\frac{3}{2}e_{q'} + a_{8a}(BM_1,M_2)\right)(\bar{q}q')_{S+P}\otimes(\bar{q}'b)_{S-P} \\
&+ \quad a_9^p(BM_1,M_2)\frac{3}{2}e_{q'}(\bar{q}'q')_{V-A}\otimes(\bar{q}b)_{V-A} \\
&+ \quad \left(a_{10}^p(BM_1,M_2)\frac{3}{2}e_{q'} + a_{10a}^p(BM_1,M_2)\right)(\bar{q}q')_{V-A}\otimes(\bar{q}'b)_{V-A}\Big\},
\end{aligned} \tag{7}
$$

where the symbol $\otimes$ denotes $\langle M_1M_2|j_2\otimes j_1|B\rangle \equiv \langle M_2|j_2|0\rangle\langle M_1|j_1|B\rangle$. M_1 represents the meson which picks up the spectator quark through this paper. For M_1 a light *vector* meson and M_2 a light *pseudoscalar* meson, the effective a_i^p's which contain next-to-leading order(NLO) coefficients and $\mathcal{O}(\alpha_s)$ hard scattering corrections are found to be

$$
\begin{aligned}
a_{1,2}^c(BV,P) &= 0, \quad a_i^c(BV,P) = a_i^u(BV,P) \equiv a_i(BV,P), i = 3,5,7,8,9,10,8a,10a. \\
a_1^u(BV,P) &= C_1 + \frac{C_2}{N} + \frac{\alpha_s}{4\pi}\frac{C_F}{N}C_2F_P, \\
a_2^u(BV,P) &= C_2 + \frac{C_1}{N} + \frac{\alpha_s}{4\pi}\frac{C_F}{N}C_1F_P, \\
a_3(BV,P) &= C_3 + \frac{C_4}{N} + \frac{\alpha_s}{4\pi}\frac{C_F}{N}C_4F_P, \\
a_4^p(BV,P) &= C_4 + \frac{C_3}{N} + \frac{\alpha_s}{4\pi}\frac{C_F}{N}\Bigg[C_3(F_P + G_P(s_q) + G_P(s_b)) + C_1G_P(s_p) \\
&\qquad +(C_4+C_6)\sum_{f=u}^{b}G_P(s_f) + C_gG_{P,g}\Bigg], \\
a_5(BV,P) &= C_5 + \frac{C_6}{N} + \frac{\alpha_s}{4\pi}\frac{C_F}{N}C_6(-F_P - 12), \\
a_6^p(BV,P) &= C_6 + \frac{C_5}{N} + \frac{\alpha_s}{4\pi}\frac{C_F}{N}\Bigg[C_1G'_P(s_p) + C_3(G'_P(s_q) + G'_P(s_b)) \\
&\qquad +(C_4+C_6)\sum_{f=u}^{b}G'_P(s_f) + C_gG'_{P,g}\Bigg], \\
a_7(BV,P) &= C_7 + \frac{C_8}{N} + \frac{\alpha_s}{4\pi}\frac{C_F}{N}C_8(-F_P - 12), \\
a_8(BV,P) &= C_8 + \frac{C_7}{N}, \\
a_{8a}(BV,P) &= \frac{\alpha_s}{4\pi}\frac{C_F}{N}\left[(C_8+C_{10})\sum_{f=u}^{b}\frac{3}{2}e_fG'_P(s_f) + C_9\frac{3}{2}(e_qG'_P(s_q) + e_bG'_P(s_b))\right], \\
a_9(BV,P) &= C_9 + \frac{C_{10}}{N} + \frac{\alpha_s}{4\pi}\frac{C_F}{N}C_{10}F_P,
\end{aligned}
$$

$$
\begin{aligned}
a_{10}(BV,P) &= C_{10}+\frac{C_9}{N}+\frac{\alpha_s}{4\pi}\frac{C_F}{N}C_9F_P, \\
a_{10a}(BV,P) &= \frac{\alpha_s}{4\pi}\frac{C_F}{N}\left[(C_8+C_{10})\frac{3}{2}\sum_{f=u}^{b}e_fG_P(s_f)+C_9\frac{3}{2}(e_qG_P(s_q)+e_bG_P(s_b))\right], \qquad (8)
\end{aligned}
$$

where $p=(u,c)$, $q=(d,s)$, $q'=(u,d,s)$, and $f=(u,d,s,c,b)$. $C_F=(N^2-1)/(2N)$ and $N=3$ is the number of colors. The internal quark mass in the penguin diagrams enters as $s_f=m_f^2/m_b^2$. $\bar{x}=1-x$ and $\bar{u}=1-u$.

$$
\begin{aligned}
F_P &= -12\ln\frac{\mu}{m_b}-18+f_P^I+f_P^{II}, \qquad (9)\\
f_P^I &= \int_0^1 dx g(x)\phi_P(x), \qquad g(x)=3\frac{1-2x}{1-x}\ln x-3i\pi,\\
f_P^{II} &= \frac{4\pi^2}{N}\frac{f_Vf_B}{A_0^V(0)M_B^2}\int_0^1 dz\frac{\phi_B(z)}{z}\int_0^1 dx\frac{\phi_V(x)}{x}\int_0^1 dy\frac{\phi_P(y)}{y}, \qquad (10)\\
G_{P,g} &= -\int_0^1 dx\frac{2}{\bar{x}}\phi_P(x), \qquad (11)\\
G_P(s_q) &= \frac{2}{3}-\frac{4}{3}\ln\frac{\mu}{m_b}+4\int_0^1 dx\phi_P(x)\int_0^1 du \quad u\bar{u}\ln\left[s_q-u\bar{u}\bar{x}-i\epsilon\right], \qquad (12)\\
G'_{P,g} &= -\int_0^1 dx\frac{3}{2}\phi_P^0(x)=-\frac{3}{2}, \qquad (13)\\
G'_P(s_q) &= \frac{1}{3}-\ln\frac{\mu}{m_b}+3\int_0^1 dx\phi_P^0(x)\int_0^1 du \quad u\bar{u}\ln\left[s_q-u\bar{u}\bar{x}-i\epsilon\right], \qquad (14)
\end{aligned}
$$

For M_1 is *pseudoscalar* and M_2 is *vector*, the co-efficents are

$$
\begin{aligned}
a_{1,2}^c(BP,V) &= 0, \quad a_i^c(BP,V)=a_i^u(BP,V)\equiv a_i(BP,V), i=3,5,6,7,8,9,10,10a.\\
a_1^u(BP,V) &= C_1+\frac{C_2}{N}+\frac{\alpha_s}{4\pi}\frac{C_F}{N}C_2F_V,\\
a_2^u(BP,V) &= C_2+\frac{C_1}{N}+\frac{\alpha_s}{4\pi}\frac{C_F}{N}C_1F_V,\\
a_3(BP,V) &= C_3+\frac{C_4}{N}+\frac{\alpha_s}{4\pi}\frac{C_F}{N}C_4F_V,\\
a_4^p(BP,V) &= C_4+\frac{C_3}{N}+\frac{\alpha_s}{4\pi}\frac{C_F}{N}\Big[C_3(F_V+G_V(s_q)+G_V(s_b))+C_1G_V(s_p)\\
&\qquad +(C_4+C_6)\sum_{f=u}^{b}G_V(s_f)+C_gG_{V,g}\Big],\\
a_5(BP,V) &= C_5+\frac{C_6}{N}+\frac{\alpha_s}{4\pi}\frac{C_F}{N}C_6(-F_V-12),\\
a_6(BP,V) &= C_6+\frac{C_5}{N},\\
a_7(BP,V) &= C_7+\frac{C_8}{N}+\frac{\alpha_s}{4\pi}\frac{C_F}{N}C_8(-F_V-12),
\end{aligned}
$$

$$
\begin{aligned}
a_8(BP,V) &= C_8 + \frac{C_7}{N}, \\
a_9(BP,V) &= C_9 + \frac{C_{10}}{N} + \frac{\alpha_s}{4\pi}\frac{C_F}{N}C_{10}F_V, \\
a_{10}(BP,V) &= C_{10} + \frac{C_9}{N} + \frac{\alpha_s}{4\pi}\frac{C_F}{N}C_9F_V, \\
a_{10a}(BP,V) &= \frac{\alpha_s}{4\pi}\frac{C_F}{N}\left[(C_8+C_{10})\frac{3}{2}\sum_{f=u}^{b}e_fG_V(s_f) + C_9\frac{3}{2}(e_qG_V(s_q)+e_bG_V(s_b))\right], \qquad (15)
\end{aligned}
$$

where

$$
\begin{aligned}
F_V &= -12\ln\frac{\mu}{m_b} - 18 + f_V^I + f_V^{II}, \qquad (16)\\
f_V^I &= \int_0^1 dx g(x)\phi_V(x), \qquad g(x) = 3\frac{1-2x}{1-x}\ln x - 3i\pi, \\
f_V^{II} &= \frac{4\pi^2}{N}\frac{f_Pf_B}{f_+^{B\to P}(0)M_B^2}\int_0^1 dz\frac{\phi_B(z)}{z}\int_0^1 dx\frac{\phi_P(x)}{x}\int_0^1 dy\frac{\phi_V(y)}{y}, \qquad (17)\\
G_{V,g} &= -\int_0^1 dx\frac{2}{\bar{x}}\phi_V(x), \qquad (18)\\
G_V(s_q) &= \frac{2}{3} - \frac{4}{3}\ln\frac{\mu}{m_b} + 4\int_0^1 dx\phi_V(x)\int_0^1 du \quad u\bar{u}\ln\left[s_q - u\bar{u}\bar{x} - i\epsilon\right]. \qquad (19)
\end{aligned}
$$

For both M_1 and M_2 are *pseudoscalars* , the co-efficents are

$$
\begin{aligned}
a_{1,2}^c(BM_1,M_2) &= 0, \quad a_i^c(BM_1,M_2) = a_i^u(BM_1,M_2), i = 3,5,7,8,9,10,8a,10a. \\
a_1^u(BM_1,M_2) &= C_1 + \frac{C_2}{N} + \frac{\alpha_s}{4\pi}\frac{C_F}{N}C_2F_{M_2}, \\
a_2^u(BM_1,M_2) &= C_2 + \frac{C_1}{N} + \frac{\alpha_s}{4\pi}\frac{C_F}{N}C_1F_{M_2}, \\
a_3(BM_1,M_2) &= C_3 + \frac{C_4}{N} + \frac{\alpha_s}{4\pi}\frac{C_F}{N}C_4F_{M_2}, \\
a_4^p(BM_1,M_2) &= C_4 + \frac{C_3}{N} + \frac{\alpha_s}{4\pi}\frac{C_F}{N}\Bigg[C_3(F_{M_2}+G_{M_2}(s_q)+G_{M_2}(s_b)) + C_1G_{M_2}(s_p) \\
&\quad +(C_4+C_6)\sum_{f=u}^{b}G_{M_2}(s_f) + C_gG_{M_2,g}\Bigg], \\
a_5(BM_1,M_2) &= C_5 + \frac{C_6}{N} + \frac{\alpha_s}{4\pi}\frac{C_F}{N}C_6(-F_{M_2}-12), \\
a_6^p(BM_1,M_2) &= C_6 + \frac{C_5}{N} + \frac{\alpha_s}{4\pi}\frac{C_F}{N}\Bigg[C_1G'_P(s_p) + C_3(G'_{M_2}(s_q)+G'_{M_2}(s_b)) \\
&\quad +(C_4+C_6)\sum_{f=u}^{b}G'_{M_2}(s_f) + C_gG'_{M_2,g}\Bigg], \\
a_7(BM_1,M_2) &= C_7 + \frac{C_8}{N} + \frac{\alpha_s}{4\pi}\frac{C_F}{N}C_8(-F_{M_2}-12),
\end{aligned}
$$

$$
\begin{aligned}
a_8(BM_1,M_2) &= C_8+\frac{C_7}{N},\\
a_{8a}(BM_1,M_2) &= \frac{\alpha_s}{4\pi}\frac{C_F}{N}\left[(C_8+C_{10})\sum_{f=u}^{b}\frac{3}{2}e_f G'_{M_2}(s_f)+C_9\frac{3}{2}(e_q G'_{M_2}(s_q)+e_b G'_{M_2}(s_b))\right],\\
a_9(BM_1,M_2) &= C_9+\frac{C_{10}}{N}+\frac{\alpha_s}{4\pi}\frac{C_F}{N}C_{10}F_{M_2},\\
a_{10}(BM_1,M_2) &= C_{10}+\frac{C_9}{N}+\frac{\alpha_s}{4\pi}\frac{C_F}{N}C_9F_{M_2},\\
a_{10a}(BM_1,M_2) &= \frac{\alpha_s}{4\pi}\frac{C_F}{N}\left[(C_8+C_{10})\frac{3}{2}\sum_{f=u}^{b}e_f G_{M_2}(s_f)+C_9\frac{3}{2}(e_q G_{M_2}(s_q)+e_b G_{M_2}(s_b))\right],
\end{aligned}
\tag{20}
$$

where

$$F_{M_2} = -12\ln\frac{\mu}{m_b}-18+f^I_{M_2}+f^{II}_{M_2}, \tag{21}$$

$$f^I_{M_2} = \int_0^1 dx g(x)\phi_{M_2}(x), \qquad g(x)=3\frac{1-2x}{1-x}\ln x-3i\pi,$$

$$f^{II}_{M_2} = \frac{4\pi^2}{N}\frac{f_{M_1}f_B}{f_+^{B\to M_1}(0)M_B^2}\int_0^1 dz\frac{\phi_B(z)}{z}\int_0^1 dx\frac{\phi_{M_1}(x)}{x}\int_0^1 dy\frac{\phi_{M_2}(y)}{y}, \tag{22}$$

$$G_{M_2,g} = -\int_0^1 dx\frac{2}{\bar{x}}\phi_{M_2}(x), \tag{23}$$

$$G_{M_2}(s_q) = \frac{2}{3}-\frac{4}{3}\ln\frac{\mu}{m_b}+4\int_0^1 dx\phi_{M_2}(x)\int_0^1 du \quad u\bar{u}\ln\left[s_q-u\bar{u}\bar{x}-i\epsilon\right], \tag{24}$$

$$G'_{M_2,g} = -\int_0^1 dx\frac{3}{2}\phi^0_{M_2}(x)=-\frac{3}{2}, \tag{25}$$

$$G'_{M_2}(s_q) = \frac{1}{3}-\ln\frac{\mu}{m_b}+3\int_0^1 dx\phi^0_{M_2}(x)\int_0^1 du \quad u\bar{u}\ln\left[s_q-u\bar{u}\bar{x}-i\epsilon\right], \tag{26}$$

where $\phi_P(x)$ and $\phi^0_P(x)$ are the pseudoscalar meson's twist-2 and twist-3 distribution amplitudes (DA) respectively. $\phi_V(x)=\phi_{V,\parallel}(x)$ is the leading twist DA for the longitudinally polarized vector meson states. We have used the fact that light vector meson is longitudinally polarized in $B\to PV$ decays. In the derivation of the effective coefficients a_i's we have used NDR scheme and assumption of asymptotic DAs. The infrared divergences in $Fig.1.a-d$ are cancelled in their sum.

With the effective coefficients in Eqs.8, 15 and 20 we can write down the decay amplitudes of the decay modes (we only list four decay modes here which have been observed by CLEO. The other decay modes are given in appendix A)

$$\mathcal{M}(B^+\to\eta' K^+) = i\frac{G_F}{\sqrt{2}}f_K(m_B^2-m_{\eta'}^2)F^{B\to\eta'}(m_K^2)V_{cb}(1-\frac{\lambda^2}{2})\Big\{R_c e^{i\gamma}\Big[a_1(X)+a_4^u(X)$$

$$
\begin{aligned}
&+R_K[a_6^u(X)+a_8(X)+a_{8a}(X)]+a_{10}(X)+a_{10a}(X)\Big]\\
&+a_4^c(X)+a_{10}(X)+a_{10a}(X)+R_K[a_6^c(X)+a_8(X)+a_{8a}(X)]\Big\}\\
&+i\frac{G_F}{\sqrt{2}}(m_B^2-m_K^2)F^{B\to K}(m_{\eta'}^2)V_{cb}(1-\frac{\lambda^2}{2})\Big\{R_c e^{i\gamma}\Big[a_2(Y)f_{\eta'}^u\\
&+(a_3(Y)-a_5(Y))(2f_{\eta'}^u+f_{\eta'}^s)+a_4^u(Y)f_{\eta'}^s+R_{\eta'}^s(a_6^u(Y)\\
&-\frac{1}{2}a_8(Y)+a_{8a}(Y))(f_{\eta'}^s-f_{\eta'}^u)+\frac{1}{2}(-a_7(Y)+a_9(Y))(f_{\eta'}^u-f_{\eta'}^s)\\
&+(-\frac{1}{2}a_{10}(Y)+a_{10a}(Y))f_{\eta'}^s\Big]+(a_3(Y)-a_5(Y))(2f_{\eta'}^u+f_{\eta'}^s)+a_4^c(Y)f_{\eta'}^s\\
&+R_{\eta'}^s(a_6^c(Y)-\frac{1}{2}a_8(Y)+a_{8a}(Y))(f_{\eta'}^s-f_{\eta'}^u)\\
&+\frac{1}{2}(a_9(Y)+a_7(Y))(f_{\eta'}^u-f_{\eta'}^s)-(\frac{1}{2}a_{10}(Y)-a_{10a}(Y))f_{\eta'}^s\Big\}. \qquad (27)\\
&with\ X=B^-\eta',K^-\ \ and\ \ Y=B^-K^-,\eta'.
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}(B^0\to\eta' K^0) &= i\frac{G_F}{\sqrt{2}}(m_B^2-m_K^2)F^{B\to K}(m_{\eta'}^2)V_{cb}(1-\frac{\lambda^2}{2})\Big\{R_c e^{i\gamma}\Big[a_2(X)f_{\eta'}^u+a_3(X)(2f_{\eta'}^u+f_{\eta'}^s)\\
&+ a_4^u(X)f_{\eta'}^s-a_5(X)(2f_{\eta'}^u+f_{\eta'}^s)+R_{\eta'}^s(a_6^u(X)-\frac{1}{2}a_8(X)+a_{8a}(X))(f_{\eta'}^s-f_{\eta'}^u)\\
&+ \frac{1}{2}(-a_7(X)+a_9(X))(f_{\eta'}^u-f_{\eta'}^s)+(-\frac{1}{2}a_{10}(X)+a_{10a}(X))f_{\eta'}^s\Big]\\
&+ \Big[a_3(X)(2f_{\eta'}^u+f_{\eta'}^s)+a_4^c(X)f_{\eta'}^s-a_5(X)(2f_{\eta'}^u+f_{\eta'}^s)\\
&+ R_{\eta'}^s(a_6^c(X)-\frac{1}{2}a_8(X)+a_{8a}(X))(f_{\eta'}^s-f_{\eta'}^u)\\
&+ \frac{1}{2}(a_9(X)-a_7(X)+)(f_{\eta'}^s-f_{\eta'}^u)+(-\frac{1}{2}a_{10}(X)+a_{10a}(X))f_{\eta'}^s\Big]\Big\}\\
&+ i\frac{G_F}{\sqrt{2}}f_K(m_B^2-m_{\eta'}^2)F^{B\to\eta'}(m_K^2)V_{cb}(1-\frac{\lambda^2}{2})\Big\{R_c e^{i\gamma}\\
&\quad \Big[a_4^u(Y)+R_K(a_6^u(Y)-\frac{1}{2}a_8(Y)+a_{8a}(Y))-\frac{1}{2}a_{10}(Y)+a_{10a}(Y)\Big]\\
&+ a_4^c(Y)+R_K(a_6^c(Y)-\frac{1}{2}a_8(Y)+a_{8a}(Y))-\frac{1}{2}a_{10}(Y)+a_{10a}(Y)\Big\}, \qquad (28)\\
&\quad with\ X=B^0K^0,\eta'\ \ and\ \ Y=B^0\eta',K^0.
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}(B^+\to\eta K^{*+}) &= \frac{G_F}{\sqrt{2}}A_0^{B\to K^*}(m_\eta^2)m_B^2V_{cb}(1-\frac{\lambda^2}{2})\Big\{R_c e^{i\gamma}\Big[a_2(X)f_\eta^u+a_3(X)(2f_\eta^u+f_\eta^s)\\
&+ a_4^u(X)f_\eta^s-a_5(X)(2f_\eta^u+f_\eta^s)-R_\eta^s(a_6^u(X)-\frac{1}{2}a_8(X)+a_{8a}(X))(f_\eta^s-f_\eta^u)\\
&+ \frac{1}{2}(a_9(X)-a_7(X))(f_\eta^u-f_\eta^s)-(\frac{1}{2}a_{10}(X)+a_{10a}(X))f_\eta^s\Big]+a_3(X)(2f_\eta^u+f_\eta^s)
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad a_4^c(X)f_\eta^s - a_5(X)(2f_\eta^u + f_\eta^s) - R_\eta^s(a_6^c(X) - \frac{1}{2}a_8(X) + a_{8a}(X))(f_\eta^s - f_\eta^u) \\
&+ \quad \frac{1}{2}(a_9(X) - a_7(X))(f_\eta^u - f_\eta^s) - (\frac{1}{2}a_{10}(X) + a_{10a}(X))f_\eta^s\Big\} \\
&+ \quad \frac{G_F}{\sqrt{2}}F^{B\to\eta}f_{K^*}m_B^2V_{cb}(1-\frac{\lambda^2}{2})\Big\{R_ce^{i\gamma}\left[a_1(Y) + a_4^u(Y)\right. \\
&+ \quad \left. a_{10}(Y) + a_{10a}(Y)\right] + a_4^c(Y) + a_{10}(Y) + a_{10a}(Y)\Big\}, \qquad (29) \\
&\quad\quad with\ X = B^-K^{*-}, \eta'\ and\ Y = B^-\eta', K^{*-}.
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}(B^0 \to \eta K^{*0}) \;=\;& \frac{G_F}{\sqrt{2}}A_0^{B\to K^*}(m_\eta^2)m_B^2V_{cb}(1-\frac{\lambda^2}{2})\Big\{R_ce^{i\gamma}\Big[a_2(X)f_\eta^u + a_3(X)(2f_\eta^u + f_\eta^s) \\
&+ \quad a_4^u(X)f_\eta^s - a_5(X)(2f_\eta^u + f_\eta^s) - R_\eta^s(a_6^u(X) - \frac{1}{2}a_8(X) + a_{8a}(X))(f_\eta^s - f_\eta^u) \\
&+ \quad \frac{1}{2}(a_9(X) - a_7(X))(f_\eta^u - f_\eta^s) - (\frac{1}{2}a_{10}(X) + a_{10a}(X))f_\eta^s\Big] + a_3(X)(2f_\eta^u + f_\eta^s) \\
&+ \quad a_c^4(X)f_\eta^s - a_5(X)(2f_\eta^u + f_\eta^s) - R_\eta^s(a_6^c(X) - \frac{1}{2}a_8(X) + a_{8a}(X))(f_\eta^s - f_\eta^u) \\
&+ \quad \frac{1}{2}(a_9(X) - a_7(X))(f_\eta^u - f_\eta^s) - (\frac{1}{2}a_{10}(X) + a_{10a}(X))f_\eta^s\Big\} \\
&+ \quad \frac{G_F}{\sqrt{2}}F^{B\to\eta}f_{K^*}m_B^2V_{cb}(1-\frac{\lambda^2}{2})\Big\{R_ce^{i\gamma}\Big[a_4^u(Y) - \frac{1}{2}a_{10}(Y) + a_{10a}(Y)\Big] \\
&+ \quad a_4^c(Y) - \frac{1}{2}a_{10}(Y) + a_{10a}(Y)\Big\}, \qquad (30) \\
&\quad\quad with\ X = B^0K^*, \eta'\ and\ Y = B^0\eta', K^{*0}.
\end{aligned}
$$

Where $R_c = \mid \frac{V_{us}V_{ub}^*}{V_{cs}V_{cb}^*} \mid = \frac{\lambda}{1-\lambda^2/2}|\frac{V_{ub}^*}{V_{cb}}|$. V_{cb}, V_{us} and V_{cs} are chosen to be real and γ is the phase of V_{ub}^*. $\lambda = |V_{us}| = 0.2196$. We will present inputs and numerical results for those magnitudes in Sec.4.

3 Re-examination of the spectator-hard-scattering mechanism

3.1 Calculation of $g^*g^* - \eta^{(\prime)}$ transition form factor $F_{g^*g^*-\eta^{(\prime)}}(Q_1^2, Q_2^2)$

Recently Ref.[19] studied $g^*g - \eta^{(\prime)}$ transition form factor $F_{g^*g-\eta^{(\prime)}}(Q_1^2, Q_2^2 = 0)$. The same method can be easily used to calculate $F_{g^*g^*-\eta^{(\prime)}}(Q_1^2, Q_2^2)$, where both of the two gluons are off-

shell. To keep the completeness of this paper we recapitulate the main points of this calculation here.

The η' meson is mainly a flavor singlet meson. Because of its singlet structure the η' meson may have gluonic content through the QCD anomaly. The contribution of gluonic wave function to the transition form factor $F_{g^*g-\eta^{(\prime)}}(Q_1^2, Q_2^2 = 0)$ has been tested, which is very small [19]. By analysing the gluonic wave function, when both of the two gluons are off-shell, it will give an extra scale suppression. So for the case of $F_{g^*g^*-\eta^{(\prime)}}(Q_1^2, Q_2^2)$, it may be safe to neglect the contribution of gluonic wave function of η' meson.

We take the $q\bar{q} - s\bar{s}$ mixing scheme for the $\eta^{(\prime)}$ meson in this calculation. Here $q\bar{q}$ means the light quark pair $u\bar{u}$ and $d\bar{d}$ [8]. In this mixing scheme the parton Fock state decomposition can be expressed as

$$\begin{aligned} \mid \eta\rangle &= cos\phi \mid \eta_q\rangle - sin\phi \mid \eta_s\rangle, \\ \mid \eta'\rangle &= sin\phi \mid \eta_q\rangle + cos\phi \mid \eta_s\rangle, \end{aligned} \tag{31}$$

where ϕ is the mixing angle, and $\mid \eta_q\rangle \sim \frac{1}{\sqrt{2}} \mid u\bar{u} + d\bar{d}\rangle$, $\mid \eta_s\rangle \sim\mid s\bar{s}\rangle$.

The diagrams contributing to the transition form factor $F_{g^*g^*-\eta^{(\prime)}}(Q_1^2, Q_2^2)$ are shown in Fig.3. By direct calculation of these two diagrams the $g^*g^* - \eta^{(\prime)}$ transition form factors can be parameterized as

$$< g_a^* g_b^* | \eta^{(\prime)} >= -4\pi\alpha_s \delta_{ab} i \epsilon_{\mu\nu\alpha\beta} Q_1^\alpha Q_2^\beta F_{g^*g^*-\eta^{(\prime)}}(Q_1^2, Q_2^2). \tag{32}$$

and $F_{g^*g^*-\eta^{(\prime)}}(Q_1^2, Q_2^2)$ is found to be

$$F_{g^*g^*-\eta^{(\prime)}}(Q_1^2, Q_2^2) = \frac{1}{2N} \sum_{q=u,d,s} f^q_{\eta^{(\prime)}} \int_0^1 dx \frac{\phi_{\eta^{(\prime)}}(x)}{(1-x)Q_1^2 + xQ_2^2 - x(1-x)m^2_{\eta^{(\prime)}} + i\epsilon} + (x \to 1-x), \tag{33}$$

where the variables $f^q_{\eta^{(\prime)}}$ can be related to the decay constants of $\mid \eta_q\rangle$ and $\mid \eta_s\rangle$

$$\begin{aligned} f^u_{\eta'} &= \frac{f_q}{\sqrt{2}} \sin\phi, \qquad f^s_{\eta'} = f_s \cos\phi, \\ f^u_{\eta} &= \frac{f_q}{\sqrt{2}} \cos\phi, \qquad f^s_{\eta} = -f_s \sin\phi. \end{aligned} \tag{34}$$

The decay constants f_q, f_s and the mixing angle ϕ have been constrained from the available experimental data, $f_q = (1.07 \pm 0.02) f_\pi$, $f_s = (1.34 \pm 0.06) f_\pi$, $\phi = 39.3^0 \pm 1.0^0$ [8].

To the accuracy of this paper, $\phi_{\eta^{(\prime)}}(x)$ is taken to be the leading twist DAs as [18] $\phi_{\eta^{(\prime)}}(x) = 6x(1-x)$.

The transition form factor will play a pivotal role in estimations of gluonic exclusive production of η' and η.

3.2 The magnitude of spectator-hard scattering mechanism for $B \to \eta' M$

In this subsection, we would re-calculate the magnitude of spectator hard scattering mechanism(SHSM) for $B \to \eta' K$ proposed in Ref.[5] and generalize it to $B \to \eta^{(\prime)} M$ where M is a light pseudoscalar or vector meson.

The SHSM is described by the Feynman diagrams in Fig.3 where b quark decays to s(d) quark and a *hard* gluon. Since the virtuality of the gluon is much larger than the typical scale of QCD boundstate Λ_{QCD} *i.e.*, $1/Q_1^2 << 1/\Lambda_{QCD}^2$, it would fluctuate into a small size fast flying color-octet $\bar{q}q$ pair well before it flys out of the light cloud of B, so it would hard interact with the spectator. More arguments for validity of using perturbative QCD in this calculation could be found in Ref.[5].

For M a light pseudoscalar, the amplitudes M_1 for Fig.3.a and M_2 for Fig.3.b are calculated to be

$$M1 = \frac{G_F}{\sqrt{2}}(-V_{tq}^* V_{tb}) C_g m_b f_B f_P (2 f_{\eta^{(\prime)}}^u + f_{\eta^{(\prime)}}^s) \alpha_s^2 \frac{i}{N_c^3} \times (m_B F_{tw2} - \mu_P F_{tw3}), \tag{35}$$

$$F_{tw2}^{\eta^{(\prime)}} = -8 \int_0^1 dz dy \phi_B(z) \phi_P^{as}(y) \frac{1-z}{y-z} \left(\frac{(Q_1 \cdot Q_2)^2}{Q_1^2 (Q_2^2 + i\epsilon)} - 1 \right) \int_0^1 dx \frac{\phi^{as}(x)}{\bar{x} Q_1^2 + x Q_2^2 - x\bar{x} m_{\eta^{(\prime)}}^2 + i\epsilon}. \tag{36}$$

$$F_{tw3}^{\eta^{(\prime)}} = -8 \int_0^1 dz dy \phi_B(z) \phi_P^0(y) \frac{1-y}{y-z} \left(\frac{(Q_1 \cdot Q_2)^2}{Q_1^2 (Q_2^2 + i\epsilon)} - 1 \right) \int_0^1 dx \frac{\phi^{as}(x)}{\bar{x} Q_1^2 + x Q_2^2 - x\bar{x} m_{\eta^{(\prime)}}^2 + i\epsilon}, \tag{37}$$

$$M2 = \frac{G_F}{\sqrt{2}} \sum_f V_{fq}^* V_{fb} C_1^f f_B f_P (2 f_{\eta^{(\prime)}}^u + f_{\eta^{(\prime)}}^s) \alpha_s^2 \frac{i}{N_c^3} \times F_{PP}^{\eta^{(\prime)}}(s_f), \tag{38}$$

$$F_{PP}^{\eta^{(\prime)}}(s_f) = 16 \int_0^1 dz dy \phi_B(z) \phi_P^{as}(y) \frac{1}{y-z} \left(Q_1^2 - \frac{(Q_1 \cdot Q_2)^2}{Q_2^2} \right)$$

$$\int_0^1 du u\bar{u}\left(-1+2\ln\frac{\mu}{m_b}-\ln(s_f-u\bar{u}\frac{Q_1^2}{m_b^2}-i\epsilon)\right)$$
$$\int_0^1 dx\frac{\phi^{as}_{\eta^{(\prime)}}(x)}{\bar{x}Q_1^2+xQ_2^2-x\bar{x}m^2_{\eta^{(\prime)}}+i\epsilon}. \tag{39}$$

When M is a light vector meson, the amplitudes M_3 and M_4 for Fig.3.a and Fig.3.b respectively are

$$\begin{aligned} M3 &= \frac{G_F}{\sqrt{2}}(-V^*_{tq}V_{tb})C_g m_b f_B f_V(2f^u_{\eta^{(\prime)}}+f^s_{\eta^{(\prime)}})\alpha_s^2\frac{1}{N_c^3}m_B\times F^{\eta^{(\prime)}}_{BPV1}, &(40)\\ F^{\eta^{(\prime)}}_{BPV1} &= -8\int_0^1 dzdy\phi_B(z)\phi^{as}_V(y)\frac{1-z}{y-z}\left(\frac{(Q_1\cdot Q_2)^2}{Q_1^2(Q_2^2+i\epsilon)}-1\right)\\ &\quad\times\int_0^1 dx\frac{\phi^{as}(x)}{\bar{x}Q_1^2+xQ_2^2-x\bar{x}m^2_{\eta^{(\prime)}}+i\epsilon}. &(41)\\ M4 &= \frac{G_F}{\sqrt{2}}\sum_f V^*_{fq}V_{fb}C_1^f f_B f_V(2f^u_{\eta^{(\prime)}}+f^s_{\eta^{(\prime)}})\alpha_s^2\frac{1}{N_c^3}\times F^{\eta^{(\prime)}}_{BPV2}(s_f), &(42)\\ F^{\eta^{(\prime)}}_{BPV2}(s_f) &= 16\int_0^1 dz\int_0^1 dy\phi_B(z)\phi^{as}_V(y)\frac{1}{y-z}\left(Q_1^2-\frac{(Q_1\cdot Q_2)^2}{Q_2^2}\right)\\ &\quad\times\int_0^1 du u\bar{u}\left(-1+2\ln\frac{\mu}{m_b}-\ln(s_f-u\bar{u}\frac{Q_1^2}{m_b^2}-i\epsilon)\right)\\ &\quad\times\int_0^1 dx\frac{\phi^{as}_{\eta^{(\prime)}}(x)}{\bar{x}Q_1^2+xQ_2^2-x\bar{x}m^2_{\eta^{(\prime)}}+i\epsilon}. &(43)\end{aligned}$$

Where $q=d,s$ and $f=u,c$. $Q_1=(1-z)P_B-(1-y)P_M$ and $Q_2=zP_B-yP_M$.

It should be noted that the effective vertex $b\to sg^*$ calculated from full theory in Ref.[20] could not be used here, otherwise the contribution of top quark penguin will be double counted when amplitudes of SHSM are added to these in Eq.8. Other four fermion operators, say, O_3, O_4 and O_6 can also be inserted in Fig.3. However, dominate contributions come from the insertion of $O_1^{u,c}$.

With note of

$$\frac{(Q_1\cdot Q_2)^2}{Q_1^2Q_2^2}-1\simeq-\frac{1}{4z(1-z)},\quad Q_1^2-\frac{(Q_1\cdot Q_2)^2}{Q_2^2}\simeq m_B^2\frac{(y-z)}{4z}$$

and the DAs to be given in next section, we can see that $F^{\eta^{(\prime)}}_{tw2}$, $F^{\eta^{(\prime)}}_{PP}(s_f)$, $F^{\eta^{(\prime)}}_{BPV1}$, and $F^{\eta^{(\prime)}}_{BPV2}(s_f)$ are free of IR divergence. Since the twist-3 DA $\phi^0_P(y)$ for pseudoscalar meson does not fall at end point, integration of $1/(y-z)$ over y results in $\ln(z)$, but such large logarithm will be smeared by DA of B meson through the integration $\int\phi_B(z)\ln z dz$, which are shown by analytic expressions in Appendix.B. Generally the integrations are free of IR divergence.

4 Numerical calculations and discussions of results

In the numerical calculations we use [21]

$$\tau(B^+) = 1.65 \times 10^{-12}s, \quad \tau(B^0) = 1.56 \times 10^{-12}s,$$

$$M_B = 5.2792GeV, \quad m_b = 4.8GeV, \quad m_c = 1.4GeV,$$

$$m_s = 80MeV, \quad f_B = 0.180GeV, \quad f_\pi = 0.133GeV,$$

$$f_K = 0.158GeV, \quad f_{K^*} = 0.214GeV, \quad f_\rho = 0.21GeV.$$

For the chiral enhancement factors for the pseudoscalar mesons, we take

$$R_{\pi^\pm} = R_{K^{\pm,0}} \simeq 1.2,$$

which are consistent with the values used in [14, 22, 23], and

$$R^s_{\eta^{(\prime)}} = \frac{m^2_{\eta^{(\prime)}}}{m_s m_b}, \quad \mu_P = \frac{m_b R_P}{2}.$$

We take the leading-twist DA $\phi(x)$ and the twist-3 DA $\phi^0(x)$ of light pseudoscalar and vector mesons as the asymptotic form [24]

$$\phi_{P,V}(x) = 6x(1-x), \quad \phi^0_P(x) = 1. \tag{44}$$

For the B meson, its DA is modeled as [25, 26],

$$\phi_B(x) = N_B x^2(1-x)^2 \exp\left[-\frac{M_B^2\ x^2}{2\omega_B^2}\right], \tag{45}$$

with $\omega_B = 0.3$ GeV, and N_B is the normalization constant to make $\int_0^1 dx\phi_B(x) = 1$. $\phi_B(x)$ is strongly peaked around $x = 0.075$, which is consistent with the observation of Heavy Quark Effective Theory that the wave function should be peaked around Λ_{QCD}/M_B. We get the object $\int_0^1 dx\phi_B(x)/x = 14.62$ which is very near to the argument of $\int_0^1 dx\phi_B(x)/x \simeq m_B/\lambda_B = 17.6$ in Ref.[14].

We have used the unitarity of the CKM matrix $V^*_{uq}V_{ub} + V^*_{cq}V_{cb} + V^*_{tq}V_{tb} = 0$ to decompose the amplitudes into terms containing $V^*_{uq}V_{ub}$ and $V^*_{cq}V_{cb}$, and

$$\begin{aligned} &|V_{us}| = \lambda = 0.2196, \quad &&|V_{ub}/V_{cb}| = 0.085 \pm 0.02, \\ &|V_{cs}| = 1 - \lambda^2/2 &&|V_{ud}| = 1 - \lambda^2/2. \end{aligned} \tag{46}$$

We would use the latest CLEO results for $|V_{cb}|$[27, 28]

$$|V_{cb}| = 0.0464 \pm 0.0020(stat.) \pm 0.0021(syst.) \pm 0.0021(theor.), \tag{47}$$

and leave the CKM angle γ as a free parameter.

For the form factors for $B \to \pi, K, K^*$ and ρ, we take the results of light-cone sum rule [29, 30]

$$F^{B\to\pi^\pm}(0) = 0.3, \quad F^{B\to K}(0) = 0.36, \quad A_0^{B\to\rho^\pm}(0) = 0.372, \quad A_0^{B\to K^*}(0) = 0.470. \tag{48}$$

Compared with these rather well studied form factors, the form factors for $B \to \eta^{(\prime)}$ are poorly known, which has hindered theoretical predictions for B decays involving $\eta^{(\prime)}$ very much for a long time. Neglecting $\eta^{(\prime)}$ masses effects, we argue the following scaling relations for from factors at large recoil point in the heavy quark limit $m_b \to \infty$

$$\frac{F^{B\to\eta}(0)}{F^{B\to\pi}(0)} \simeq \frac{f_\eta^u}{f_\pi}, \qquad \frac{F^{B\to\eta'}(0)}{F^{B\to\pi}(0)} \simeq \frac{f_{\eta'}^u}{f_\pi}. \tag{49}$$

We get

$$F^{B\to\eta}(0) = 0.176, \quad F^{B\to\eta'}(0) = 0.142. \tag{50}$$

which agree well with the values $F^{B\to\eta}(0) = 0.181$ and $F^{B\to\eta'}(0) = 0.148$ in Ref.[22, 23].

Taking $\gamma = 55°$ as benchmark, we present our numerical results in Table.1. As shown in the table, for the case of $F^{B\eta^{(\prime)}}(0) = 0.176(0.142)$, our estimations of the conventional mechanism contributions to $B^+ \to K^+\eta'$ and $B^0 \to \eta' K^0$ are about $(30 \sim 40) \times 10^{-6}$ which confirm many theoretical estimations in the literature[22, 31]. It implies that significant new contributions to those decays are needed to interpret the CLEO data. Before going further, we note that the theoretical predictions for $B \to \eta K^*$ are also much smaller than their experimental results. In contrast to $B \to \eta' K$, new contributions to B decays ηK^* from both $b \to s(c\bar{c}) \to s\eta$ and SHSM are negligible. Because η and η' mesons are much heavier than π, the recoiling of $\eta^{(\prime)}$ should be smaller than that of π, so the scaling relation of Eq.49 might be broken to certain extent. Driven by the facts, we pose 30% enhancement to the form factor $F^{B\to\eta^{(\prime)}}(0)$. From column 4 of Table.I, we can see the 30% enhancement of $F^{B\to\eta^{(\prime)}}(0)$ is preferred by experimental data. However, conventional mechanism contributions to $B^+ \to \eta' K^+$ and $B^0 \to \eta' K^0$ are just enhanced by 17% and still around 1/2 of their experimental center values. With incorporation

Table 1: Branching ratios (in units of 10^{-6}) for B charmless decays involving $\eta^{(\prime)}$. Experimental results are taken from [11]. Our results are made for four cases with $\gamma = 55^\circ$ and two cases with $\gamma = 85^\circ$. The labels CM A and CM B represent estimations of the conventional mechanism contributions(CM) with inputs $F^{B\eta^{(\prime)}}(0) = 0.142(0.176)$(A) and $F^{B\eta^{(\prime)}}(0) = 0.185(0.229)$(B) respectively. Columns with CM A(B)+SH represent our results with the incorporation of spectator hard scattering mechanism.

Decay	$\gamma = 55^\circ$				$\gamma = 85^\circ$		CLEO[11]
Modes	CM A	CM A+SH	CM B	CM B+SH	CM B	CM B+SH	B or 90% UL
$B^+ \to \eta' K^+$	27.1	63.4	32.8	68.9	35.4	69.3	$80^{+10}_{-9} \pm 7$
$B^0 \to \eta' K^0$	28.4	67.1	35.0	74.8	34.5	73.1	$89^{+18}_{-16} \pm 9$
$B^+ \to \eta K^{*+}$	5.40	5.24	6.88	6.73	9.62	9.44	$26.4^{+9.6}_{-8.2} \pm 3.3$
$B^0 \to \eta K^{*0}$	7.40	7.27	9.96	9.79	9.82	9.65	$13.8^{+5.5}_{-4.6} \pm 1.6$
$B^+ \to \eta' \pi^+$	3.78	7.60	6.12	10.8	4.86	11.0	<12
$B^0 \to \eta' \pi^0$	0.13	0.51	0.16	0.52	0.20	0.74	<5.7
$B^+ \to \eta' K^{*+}$	5.87	6.40	5.02	5.47	3.92	4.21	<35
$B^0 \to \eta' K^{*0}$	3.22	3.68	2.10	2.49	2.06	2.42	<24
$B^+ \to \eta' \rho^+$	3.73	3.71	6.36	6.33	6.54	6.60	<33
$B^0 \to \eta' \rho^0$	0.020	0.010	0.023	0.012	0.033	0.020	<12
$B^+ \to \eta K^+$	3.11	7.85	1.49	6.00	1.21	4.69	<6.9
$B^0 \to \eta K^0$	1.64	8.89	0.28	8.88	0.28	7.87	<9.3
$B^+ \to \eta \pi^+$	6.54	4.84	10.2	8.02	7.96	5.15	<5.7
$B^0 \to \eta \pi^0$	0.36	0.41	0.42	0.46	0.53	0.60	<2.9
$B^+ \to \eta \rho^+$	6.45	6.39	10.7	10.6	10.3	10.4	<15
$B^0 \to \eta \rho^0$	0.02	0.017	0.023	0.001	0.033	0.015	<10

of SHSM contributions, theoretical results are improved much, and the experimental data from CLEO could be accommodated, although our results are still slightly lower than the center values. It is also noted that BaBar collaborations[32] have reported their preliminary results

$$\mathcal{B}(B^+ \to \eta' K^+) = (62 \pm 18 \pm 8) \times 10^{-6}, \;\; \mathcal{B}(B^0 \to \eta' K^0) < 112 \times 10^{-6} \tag{51}$$

where the center value of $\mathcal{B}(B^+ \to \eta' K^+)$ is smaller than that of CLEO's. It might be safe to conclude that the large branching ratio of $B \to \eta' K$ could be understood in the SM.

Topologically, SHSM would contribute to the decays $B \to \eta' K^*$ the same as it to $B \to \eta' K$. However, SHSM amplitudes depend on the spin configurations of K and K^*. As shown in $Eqs.35 - 43$, compared with the SHSM amplitudes for $B \to K^* \eta'$, SHSM amplitudes for $B \to \eta' K$ are chirally enhanced by K twist-3 DA. So that the new contributions to $B \to \eta' K$ are much larger than its to $B \to \eta' K^*$. In Fig. 4, we display our results for the branching ratios of B two-body charmless decays involving $\eta^{(\prime)}$ as functions of the weak angle γ. From Fig.4, we can see that SHSM contributions to $B \to \eta' + P$ are much larger than its contributions to $B \to \eta' + V$. It is easy to understand that SHSM contributions to $B \to \eta + M$ are very small because of cancellations between $(u\bar{u} + d\bar{d})$ and $s\bar{s}$ contents of η. For the four measured decays $B^+ \to (\eta' K^+, \; \eta K^*)$ and $B^0 \to (\eta' K^0, \eta K^{0*})$, our results generally agree with CLEO data. In our calculations, we have used large form factor $F^{B\to\eta}(0)$ to give large $\mathcal{B}(B^+ \to \eta K^{*+})$ and $\mathcal{B}(B^0 \to \eta K^{*0})$, meanwhile we meet constraint from $B^+ \to \eta \pi^+$ whose upper limit set by CLEO is $\mathcal{B}(B^+ \to \eta \pi^+) < 5.7 \times 10^{-6}$. Since $B^+ \to \eta \pi^+$ is dominated by the tree amplitude with $if_\pi F^{B\to\eta}(m_\pi^2) a_1$, $\mathcal{B}(B^+ \to \eta \pi^+)$ is very sensitive to the form factor $F^{B\to\eta}(m_\pi^2)$. To meet the large branching ratio $\mathcal{B}(B^+ \to \eta K^{*+}) = (26.4^{+9.6}_{-8.2} \pm 3.3) \times 10^{-6}$ and comply with the small upper limit $\mathcal{B}(B^+ \to \eta \pi^+) < 5.7 \times 10^{-6}$, we need $\cos\gamma < 0$ to enhance $\mathcal{B}(B^+ \to \eta K^{*+})$ and suppress $\mathcal{B}(B^+ \to \eta \pi^+)$. This situation is very similar to that in $B^0(\bar{B}^0) \to \pi^\pm \pi^\mp, \; K^\pm \pi^\mp$ and $B \to \pi\rho, \pi K^*$ decays as discussed in Refs.[22, 33, 34, 35, 36, 37] recently. It is worth to note that most recent theoretical analyses of CLEO data based on different approaches endorse, although not definitely, negative $\cos\gamma$ to some extent. However global CKM fit has given the constraint $\gamma < 90°$ at 99.6% C.L. [38, 39]. With refined measurements at running B factories BELLE and BaBar to come very soon, the following B exclusive decay modes will give strong constraints on γ

$$PP \;\; modes: \;\; B^0 \to \pi^\pm \pi^\mp, \; B^0 \to K^\pm \pi^\mp, \; B^\pm \to K^\pm \pi^0, \; B^\pm \to \eta' \pi^\pm, \; B^\pm \to \eta \pi^\pm; \tag{52}$$

$$PV \;\; modes: \quad B^0 \to \pi^{\pm}\rho^{\mp},\; B^{\pm} \to \pi^{-}\omega,\; B^{\pm} \to \pi^{\pm}\rho^0,\; B^{\pm} \to K^{\pm}\omega,\; B^{\pm} \to \eta K^{*\pm}. \quad (53)$$

Branching ratios for the above decay modes are of order of $10^{-6} \sim 10^{-5}$ which can be well studied at B factories to constrain $\cos\gamma$ tightly. If the disagreement of constraints on γ between global fit and direct model calculations becomes serious, it might imply the failure of the models employed here and in Refs.[22, 33, 34, 35, 36, 37]. Very probably theories for B hadronic decays will be tested and driven by the observations to be made at BaBar and Belle.

5 Final states interactions and CP violation

As shown by Beneke, Buchalla, Neubert and Sachrajda in Refs.[14, 15], the final states interactions in charmless B two-body decays are calculable in the QCD improved factorization framework, which turn out to be nonfactorizable corrections. The nonfactorizable corrections for the decays studied in this paper are shown in Eqs.8, 15,20. SHSM amplitudes are generally nonfactorizable and always contribute large strong phase to certain decay modes. It is worth to note that some shortcomings in the "generalized factorization" are resolved in the framework employed in this paper. Nonfactorizable effects are calculated in a rigorous way here instead of being parameterized by effective color number. Since the hard scattering kernels are convoluted with the light cone DAs of the mesons, gluon virtuality $k^2 = \bar{x}m_b^2$ in the penguin diagram Fig.1.e has well defined meaning and leaves no ambiguity as to the value of k^2, which has conventionally been treated as a free phenomenological parameter in the estimations of the strong phase generated through the Bander, Silverman and Soni(BSS) mechanism[40]. So that CP asymmetries are predicted soundly in this paper.

The direct CP asymmetry parameter is defined as

$$A_{CP}^{dir} = \frac{|\mathcal{M}(B^- \to \bar{f})|^2 - |\mathcal{M}(B^+ \to f)|^2}{|\mathcal{M}(B^+ \to f)|^2 + |\mathcal{M}(B^- \to \bar{f})|^2}, \quad (54)$$

and

$$A_{CP}^{dir} = \frac{|\mathcal{M}(\bar{B}^0 \to \bar{f})|^2 - |\mathcal{M}(B^0 \to f)|^2}{|\mathcal{M}(B^0 \to f)|^2 + |\mathcal{M}(\bar{B}^0 \to \bar{f})|^2}. \quad (55)$$

For CP-violations in $B^0(\bar{B}^0) \to \eta^{(\prime)}\pi^0, \eta^{(\prime)}\rho^0,\; B^0 - \bar{B}^0$ mixing effects should be considered. However, the branching ratio for these decay modes are very small(below 10^{-6}). We would not address CP violations in those decay modes in this paper.

Our numerical results for the direct CP violations are shown in Fig.5 as functions of γ. For $\gamma \in [50^\circ, 90^\circ]$, the direct CP-violation in $B^\pm \to \eta' K^\pm$ are found about $8\% \sim 10\%$. The recent search for direct CP violation in $B^\pm \to \eta' K^\pm$ made by CLEO Collaborations[41] has reported

$$A_{CP}^{dir}(B^\pm \to \eta' K^\pm) \sim +0.03 \pm 0.12 \tag{56}$$

which results in the 90% $C.L$ interval [-0.17, 0.23] for $A^{dir}(B^\pm \to \eta' K^\pm)$. With more and better data to come soon, the sensitivity of $A^{dir}(B^\pm \to \eta' K^\pm)$ at B factories could reach $\sim \pm 4\%$. The prospect of observing direct CP violation in $B^\pm \to \eta' K^\pm$ are expected to be quite good. The direct CP violations in $B^0 \to \eta' K^0$, ηK^* and $B^+ \to \eta K^{*+}$ are also estimated to be about few percent but smaller than $A^{dir}(B^\pm \to \eta' K^\pm)$. Considering their experimental sensitivities and/or branching ratios, prospects of observing direct CP violations in these decay modes are much weaker than in $B^\pm \to \eta' K^\pm$.

Dighe, Gronau and Rosner[42] have predicted large CP violations in $B^\pm \to \eta\pi^\pm$ and $B^\pm \to \eta'\pi^\pm$. More earlier similar conclusion could be found in Ref.[43]. For $\gamma \in [50^\circ, 90^\circ]$, We find

$$\mathcal{A}_{CP}^{dir}(B^+ \to \eta'\pi^+) \approx -50\% \sim -80\%,\ \mathcal{A}_{CP}^{dir}(B^+ \to \eta\pi^+) \approx +15\% \sim +30\%. \tag{57}$$

From Fig.5.5 and Fig.5.11, we can see that SHSM contributions could enhance $\mathcal{A}^{dir}(B^+ \to \eta^{(\prime)}\pi^+)$ very much. As shown in Fig.4.5, the branching ratio $\mathcal{B}^{dir}(B^+ \to \eta'\pi^+)$ is predicted to be of order of 10^{-6}.

It should be very promising to observe direct CP violation in $B^\pm \to \eta'\pi^\pm$, $\eta\pi^\pm$ in the near future. We also predict large $\mathcal{A}^{dir}(B^+ \to \eta K^+)$. The decay modes get a large strong phase through SHSM as shown in Fig.4.11. The strong interference between tree and penguin amplitudes leads to

$$\mathcal{A}^{dir}(B^+ \to \eta K^+) \simeq +20\% \sim +50\% \tag{58}$$

for $\gamma \in [50^\circ, 90^\circ]$.

To summarize this section, we find large direct CP violations in decays $B^\pm \to \eta' K^\pm$, $B^\pm \to \eta'\pi^\pm$, $\eta\pi^\pm$ and $B^\pm \to \eta K^\pm$.

6 Summary

With the newly developed QCD improved factorization approach[14, 15], we have studied hadronic charmless two-body decays of B_u and B_d involving η or η'. Nonfactorization effects are calculated in terms of order of $\mathcal{O}(\alpha_s)$ corrections to the leading factorizatable amplitudes. We find again that the conventional mechanism account for about one half of the decay rates of $B \to \eta' K$. Significant contributions are needed to solve the "puzzle" of *unexpected* large branching ratios.

To clarify possible sources of new significant contributions to $\mathcal{B}(B \to \eta' K)$ up to date, we display the following facts. Theoretically new contributions due to intrinsic charm content of the η' have been realized to be small. Motivated by their observation of large B decay rates to $\eta' K$[1, 11], CLEO Collaborations have made searching for B decays to $\eta_c K$[28]. It is found that there is no unexpected enhancement in the $\eta_c K$ rate. Since the $\eta_c K$ rate should also be enhanced if the $\eta' K$ rate enhanced by the intrinsic charm content of the η', the CLEO results indicate that the charm content of the η' may be not the explanation for the anomalous $\mathcal{B}(B \to \eta' K)$. Alternatively one may turn to new physics. We have known that the experimental observations of $B \to X_s \gamma$ and $B \to K^* \gamma$ agree with the SM expectations, which implies that there is no *anomalous large* new physics effects in B decays. Motivated by these considerations, we have re-examined the SHSM. Starting from calculations of the transition form-factor $g^* g^* - \eta'$, we have incorporated the new contributions to that of conventional mechanism for B decays and successfully accommodated the large rate of B decays to $\eta' K$ in the SM. We have found that the absolute strength of the amplitude of SHSM for B decays to $\eta' K$ are about $1/3 \sim 1/2$ of the absolute strength of the conventional mechanism which shows the conventional mechanism to be the dominant. As shown in detail, the SHSM is important for B decays to $\eta' K$ but not for B decays to $\eta' K^*$.

We have estimated the direct CP violations in the decays. $A^{dir}(B^+ \to \eta' K^+)$ is found to be around $+8\% \sim +10\%$ for $\gamma \in [50°, 90°]$. Due to significant contributions from SHSM, we predict

$$A^{dir}(B^{\pm} \to \eta' \pi^{\pm}) \simeq 40\% \sim 70\%. \; A^{dir}(B^{\pm} \to \eta \pi^{\pm}) \simeq 20\% \sim 40\%.$$

We also predict large direct CP violation in B decays to $\eta K^{\pm}$. Prospects for observing direct CP violations in $B^{\pm}$ decays to $\eta' K^{\pm}$, $\eta' \pi^{\pm}$, $\eta \pi^{\pm}$ and $\eta K^{\pm}$ are quite promising at the ruining B

factories BaBar and BELLE.

Note added After we finished this work, we note the work[44] by Ali and Parkhomenko which is focused on $g^*g^* - \eta'$ transistion form factor. They find that the contribution gluonic content of η' to the form factor $F_{g^*g^*\eta'}(Q_1^2, Q_2^2)$ as large as few ten percentages for small Q^2 and rather small for large Q^2. Our formfactor agree with theirs to leading terms. Additionally, in the framwork[14, 15] employed here all diagrames in Fig.1 and Fig.3 are subleadling nonfactorizable contributions, so the gluonic contributions could be neglected.

Acknowledgments

We acknowledge the Grant-in-Aid for Scientific Research on Priority Areas (Physics of CP violation with contract number 09246105 and 10140028). We thank JSPS(Japan Society for the Promotion of Science) for support. The work of Y.Y is partially supported by the US-Israel Binational Science Foundation and the Israel Science Foundation. Y.Y thanks Prof. Eilam and Prof. Gronau for many valuable discussions

Appendix A

The decay amplitudes the conventional mechanism of some of the decays in terms of the effective coefficients a_i's are presented as the followings,

$$
\begin{aligned}
\mathcal{M}(B^+ \to \eta'\pi^+) \;=\;& i\frac{G_F}{\sqrt{2}}M_B^2 F^{B\to\pi}(m^2_{\eta^{(\prime)}})\lambda V_{cb}\Bigg\{R_u e^{i\gamma}\Big[a_2(B^+\pi^+,\eta')f^u_{\eta^{(\prime)}} + [a_3(B^+\pi^+,\eta') - a_5(B^+\pi^+,\eta')](2f^u_{\eta^{(\prime)}} + f^s_{\eta^{(\prime)}}) \\
&+ a_4^u(B^+\pi^+,\eta')f^u_{\eta^{(\prime)}} + \Big[a_6^u(B^+\pi^+,\eta') - \frac{1}{2}a_8(B^+\pi^+,\eta') + a_{8a}(B^+\pi^+,\eta')\Big]R^d_{\eta^{(\prime)}}f^u_{\eta^{(\prime)}} \\
&+ \frac{1}{2}\Big[a_9(B^+\pi^+,\eta') - a_7(B^+\pi^+,\eta')\Big](f^u_{\eta^{(\prime)}} - f^s_{\eta^{(\prime)}}) + [-\frac{1}{2}a_{10}(B^+\pi^+,\eta') + a_{10a}(B^+\pi^+,\eta')]f^u_{\eta^{(\prime)}}\Big] \\
&- \Big[\Big[a_3(B^+\pi^+,\eta') - a_5(B^+\pi^+,\eta')\Big](2f^u_{\eta^{(\prime)}} + f^s_{\eta^{(\prime)}}) + a_4^c(B^+\pi^+,\eta')f^u_{\eta^{(\prime)}} + \Big[a_6^c(B^+\pi^+,\eta') \\
&- \frac{1}{2}a_8(B^+\pi^+,\eta') + a_{8a}(B^+\pi^+,\eta')\Big]R^d_{\eta^{(\prime)}}f^u_{\eta^{(\prime)}} + \Big[a_9(B^+\pi^+,\eta') - a_7(B^+\pi^+,\eta')\Big]\frac{1}{2}(f^u_{\eta^{(\prime)}} - f^s_{\eta^{(\prime)}}) \\
&+ \Big[-\frac{1}{2}a_{10}(B^+\pi^+,\eta') + a_{10a}(B^+\pi^+,\eta')\Big]f^u_{\eta^{(\prime)}}\Big]\Bigg\} \\
&+ i\frac{G_F}{\sqrt{2}}(M_B^2 - m^2_{\eta^{(\prime)}})F^{B\to\eta^{(\prime)}}(0)f_\pi\lambda V_{cb}\Bigg\{R_u e^{i\gamma}\Big[a_1(B^+\eta',\pi^+) + a_4^u(B^+\eta',\pi^+) + \Big[a_6^u(B^+\eta',\pi^+) \\
&+ a_8(B^+\eta',\pi^+) + a_{8a}(B^+\eta',\pi^+)\Big]R_\pi + a_{10}(B^+\eta',\pi^+) + a_{10a}(B^+\eta',\pi^+)\Big] - \Big[a_4^c(B^+\eta',\pi^+) \\
&+ \Big[a_6^c(B^+\eta',\pi^+) + a_8(B^+\eta',\pi^+) + a_{8a}(B^+\eta',\pi^+)\Big]R_\pi + a_{10}(B^+\eta',\pi^+) + a_{10a}(B^+\eta',\pi^+)\Big]\Bigg\}; \qquad (59)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}(B^0 \to \eta^{(\prime)}\pi^0) \;=\;& -i\frac{G_F}{2}M_B^2 F^{B\to\pi}(m^2_{\eta^{(\prime)}})\lambda V_{cb}\Bigg\{R_u e^{i\gamma}\Big[a_2(B^0\pi^0,\eta^{(\prime)})f^u_{\eta^{(\prime)}} + \Big[a_3(B^0\pi^0,\eta^{(\prime)}) - a_5(B^0\pi^0,\eta^{(\prime)})\Big](2f^u_{\eta^{(\prime)}} \\
&+ f^s_{\eta^{(\prime)}}) + a_4^u(B^0\pi^0,\eta^{(\prime)})f^u_{\eta^{(\prime)}} + \Big[a_6^u(B^0\pi^0,\eta^{(\prime)}) - \frac{1}{2}a_8(B^0\pi^0,\eta^{(\prime)}) + a_{8a}(B^0\pi^0,\eta^{(\prime)})\Big]R^d_{\eta^{(\prime)}}f^u_{\eta^{(\prime)}} \\
&+ \frac{1}{2}\Big[a_9(B^0\pi^0,\eta^{(\prime)}) - a_7(B^0\pi^0,\eta^{(\prime)})\Big](f^u_{\eta^{(\prime)}} - f^s_{\eta^{(\prime)}}) + \Big[-\frac{1}{2}a_{10}(B^0\pi^0,\eta^{(\prime)}) + a_{10a}(B^0\pi^0,\eta^{(\prime)})\Big]f^u_{\eta^{(\prime)}}\Big] - \\
&\Big[\Big[a_3(B^0\pi^0,\eta^{(\prime)}) - a_5(B^0\pi^0,\eta^{(\prime)})\Big](2f^u_{\eta^{(\prime)}} + f^s_{\eta^{(\prime)}}) + a_4^c(B^0\pi^0,\eta^{(\prime)})f^u_{\eta^{(\prime)}} \\
&+ \Big[a_6^c(B^0\pi^0,\eta^{(\prime)}) - \frac{1}{2}a_8(B^0\pi^0,\eta^{(\prime)}) + a_{8a}(B^0\pi^0,\eta^{(\prime)})\Big]R^u_{\eta^{(\prime)}}f^u_{\eta^{(\prime)}} \\
&+ \frac{1}{2}\Big[a_9(B^0\pi^0,\eta^{(\prime)}) - a_7(B^0\pi^0,\eta^{(\prime)})\Big](f^u_{\eta^{(\prime)}} - f^s_{\eta^{(\prime)}}) + \Big[-\frac{1}{2}a_{10}(B^0\pi^0,\eta^{(\prime)}) + a_{10a}(B^0\pi^0,\eta^{(\prime)})\Big]f^u_{\eta^{(\prime)}}\Big]\Bigg\} \\
&+ i\frac{G_F}{2}M_B^2 F^{B\to\eta^{(\prime)}}(0)f_\pi\lambda V_{cb}\Bigg\{R_u e^{i\gamma}\Big[a_2(B^0\eta^{(\prime)},\pi^0) - a_4^u(B^0\eta^{(\prime)},\pi^0) - \Big[a_6^u(B^0\eta^{(\prime)},\pi^0) - \frac{1}{2}a_8(B^0\eta^{(\prime)},\pi^0) \\
&+ a_{8a}(B^0\eta^{(\prime)},\pi^0)\Big]R_\pi + \frac{3}{2}\Big[a_9(B^0\eta^{(\prime)},\pi^0) - a_7(B^0\eta^{(\prime)},\pi^0)\Big] + \frac{1}{2}a_{10}(B^0\eta^{(\prime)},\pi^0) - a_{10a}(B^0\eta^{(\prime)},\pi^0)\Big] \\
&- \Big[-a_4^c(B^0\eta^{(\prime)},\pi^0) - \Big[a_6^c(B^0\eta^{(\prime)},\pi^0) - \frac{1}{2}a_8(B^0\eta^{(\prime)},\pi^0) + a_{8a}(B^0\eta^{(\prime)},\pi^0)\Big]R_\pi + \frac{3}{2}\Big[a_9(B^0\eta^{(\prime)},\pi^0) \\
&- a_7(B^0\eta^{(\prime)},\pi^0)\Big] + \frac{1}{2}a_{10}(B^0\eta^{(\prime)},\pi^0) - a_{10a}(B^0\eta^{(\prime)},\pi^0)\Big]\Bigg\}; \qquad (60)
\end{aligned}
$$

$$\mathcal{M}(B^+ \to \eta' K^{*+}) = \mathcal{M}(B^+ \to \eta K^{*+})\ (\eta \to \eta'); \qquad (61)$$

$$\mathcal{M}(B^0 \to \eta' K^{*0}) = \mathcal{M}(B^+ \to \eta K^{*0})\ (\eta \to \eta'); \qquad (62)$$

$$\mathcal{M}(B^+ \to \rho^+\eta^{(\prime)}) = \frac{G_F}{2}M_B^2 A_0^{B\to\rho}(m^2_{\eta^{(\prime)}})\lambda V_{cb}\Bigg\{R_u e^{i\gamma}\Big[a_2(B^+\rho^+,\eta^{(\prime)})f^u_{\eta^{(\prime)}} + \Big[a_3(B^+\rho^+,\eta^{(\prime)}) - a_5(B^+\rho^+,\eta^{(\prime)})\Big](2f^u_{\eta^{(\prime)}} + f^s_{\eta^{(\prime)}})$$

$$
\begin{aligned}
&+a_4^u(B^+\rho^+,\eta^{(\prime)})f^u_{\eta^{(\prime)}}-\Big[a_6^u(B^+\rho^+,\eta^{(\prime)})-\frac{1}{2}a_8(B^+\rho^+,\eta^{(\prime)})+a_{8a}(B^+\rho^+,\eta^{(\prime)})\Big]R^d_{\eta^{(\prime)}}f^u_{\eta^{(\prime)}}\\
&+\frac{1}{2}\Big[a_9(B^+\rho^+,\eta^{(\prime)})-a_7(B^+\rho^+,\eta^{(\prime)})\Big](f^u_{\eta^{(\prime)}}-f^s_{\eta^{(\prime)}})+\Big[-\frac{1}{2}a_{10}(B^+\rho^+,\eta^{(\prime)})+a_{10a}(B^+\rho^+,\eta^{(\prime)})\Big]f^u_{\eta^{(\prime)}}\Big]\\
&-\Big[\Big[a_3(B^+\rho^+,\eta^{(\prime)})-a_5(B^+\rho^+,\eta^{(\prime)})\Big](2f^u_{\eta^{(\prime)}}+f^s_{\eta^{(\prime)}})+a_4^c(B^+\rho^+,\eta^{(\prime)})f^u_{\eta^{(\prime)}}-\Big[a_6^c(B^+\rho^+,\eta^{(\prime)})\\
&-\frac{1}{2}a_8(B^+\rho^+,\eta^{(\prime)})+a_{8a}(B^+\rho^+,\eta^{(\prime)})\Big]R^d_{\eta^{(\prime)}}f^u_{\eta^{(\prime)}}+\frac{1}{2}\Big[a_9(B^+\rho^+,\eta^{(\prime)})-a_7(B^+\rho^+,\eta^{(\prime)})\Big](f^u_{\eta^{(\prime)}}-f^s_{\eta^{(\prime)}})\\
&+\Big[-\frac{1}{2}a_{10}(B^+\rho^+,\eta^{(\prime)})+a_{10a}(B^+\rho^+,\eta^{(\prime)})\Big]f^u_{\eta^{(\prime)}}\Big]\Big\}\\
&+\frac{G_F}{2}M_B^2f_\rho F^{B\to\eta^{(\prime)}}(0)\lambda V_{cb}\Big\{R_ue^{i\gamma}\Big[a_1(B^+\eta^{(\prime)},\rho^+)+a_4^u(B^+\eta^{(\prime)},\rho^+)+a_{10}(B^+\eta^{(\prime)},\rho^+)\\
&+a_{10a}(B^+\eta^{(\prime)},\rho^+)\Big]-\Big[a_4^u(B^+\eta^{(\prime)},\rho^+)+a_{10}(B^+\eta^{(\prime)},\rho^+)+a_{10a}(B^+\eta^{(\prime)},\rho^+)\Big]\Big\}; \qquad (63)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}(B^0\to\rho^0\eta^{(\prime)}) \;=\;& -\frac{G_F}{2}M_B^2A_0^{B\to\rho}(m^2_{\eta^{(\prime)}})\lambda V_{cb}\Big\{R_ue^{i\gamma}\Big[a_2(B^0\rho^0,\eta^{(\prime)})f^u_{\eta^{(\prime)}}+\Big[a_3(B^0\rho^0,\eta^{(\prime)})-a_5(B^0\rho^0,\eta^{(\prime)})\Big](2f^u_{\eta^{(\prime)}}+f^s_{\eta^{(\prime)}})\\
&+a_4^u(B^0\rho^0,\eta^{(\prime)})f^u_{\eta^{(\prime)}}-\Big[a_6^u(B^0\rho^0,\eta^{(\prime)})-\frac{1}{2}a_8(B^0\rho^0,\eta^{(\prime)})+a_{8a}(B^0\rho^0,\eta^{(\prime)})\Big]R^d_{\eta^{(\prime)}}f^u_{\eta^{(\prime)}}\\
&+\frac{1}{2}\Big[a_9(B^0\rho^0,\eta^{(\prime)})-a_7(B^0\rho^0,\eta^{(\prime)})\Big](f^u_{\eta^{(\prime)}}-f^s_{\eta^{(\prime)}})+\Big[-\frac{1}{2}a_{10}(B^0\rho^0,\eta^{(\prime)})+a_{10a}(B^0\rho^0,\eta^{(\prime)})\Big]f^u_{\eta^{(\prime)}}\Big]\\
&-\Big[(a_3(B^0\rho^0,\eta^{(\prime)})-a_5(B^0\rho^0,\eta^{(\prime)})\Big](2f^u_{\eta^{(\prime)}}+f^s_{\eta^{(\prime)}})+a_4^c(B^0\rho^0,\eta^{(\prime)})f^u_{\eta^{(\prime)}}-\Big[a_6^c(B^0\rho^0,\eta^{(\prime)})\\
&-\frac{1}{2}a_8(B^0\rho^0,\eta^{(\prime)})+a_{8a}(B^0\rho^0,\eta^{(\prime)})\Big]R^d_{\eta^{(\prime)}}f^u_{\eta^{(\prime)}}+\frac{1}{2}\Big[a_9(B^0\rho^0,\eta^{(\prime)})-a_7(B^0\rho^0,\eta^{(\prime)})\Big](f^u_{\eta^{(\prime)}}-f^s_{\eta^{(\prime)}})\\
&+\Big[-\frac{1}{2}a_{10}(B^0\rho^0,\eta^{(\prime)})+a_{10a}(B^0\rho^0,\eta^{(\prime)})\Big]f^u_{\eta^{(\prime)}}\Big]\Big\}\\
&+\frac{G_F}{2}M_B^2f_\rho F^{B\to\eta^{(\prime)}}(0)\lambda V_{cb}\Big\{R_ue^{i\gamma}\Big[a_2(B^0\eta^{(\prime)},\rho^0)-a_4^u(B^0\eta^{(\prime)},\rho^0)+\frac{3}{2}\Big[a_9(B^0\eta^{(\prime)},\rho^0)+a_7(B^0\eta^{(\prime)},\rho^0)\Big]\\
&+\frac{1}{2}a_{10}(B^0\eta^{(\prime)},\rho^0)-a_{10a}(B^0\eta^{(\prime)},\rho^0)\Big]-\Big[-a_4^c(B^0\eta^{(\prime)},\rho^0)+\frac{3}{2}[a_9(B^0\eta^{(\prime)},\rho^0)+a_7(B^0\eta^{(\prime)},\rho^0)\Big]\\
&+\frac{1}{2}a_{10}(B^0\eta^{(\prime)},\rho^0)-a_{10a}(B^0\eta^{(\prime)},\rho^0)\Big]\Big\}; \qquad (64)\\
\mathcal{M}(B^+\to\eta K^+) \;=\;& \mathcal{M}(B^+\to\eta' K^+)\ (\eta'\to\eta); \qquad (65)\\
\mathcal{M}(B^0\to\eta K^0) \;=\;& \mathcal{M}(B^+\to\eta' K^0)\ (\eta'\to\eta). \qquad (66)
\end{aligned}
$$

where $R_u=\frac{1-\lambda^2/2}{\lambda}|\frac{V_{ub}}{V_{cb}}|$, $R^d_{\eta^{(\prime)}}=\frac{M^2_{\eta^{(\prime)}}}{m_sm_b}(1-\frac{f^d_{\eta^{(\prime)}}}{f^s_{\eta^{(\prime)}}})$, and the corrections to the decays from SHSM can be read from equations in Sec.3.2.

Appendix B

The integrals $F^{\eta^{(\prime)}}_{tw2}$, $F^{\eta^{(\prime)}}_{tw3}$ and $F^{\eta^{(\prime)}}_{BPV1}$ are given by

$$
\begin{aligned}
F^{\eta^{(\prime)}}_{tw2} &= 12\int_0^1 dz\frac{\phi_B(z)}{z}Y_1(z,k_{\eta^{(\prime)}},m_{\eta^{(\prime)}}), \qquad (67)\\
F^{\eta^{(\prime)}}_{tw3} &= 2\int_0^1 dz\frac{\phi_B(z)}{z(1-z)}Y_2(z,k_{\eta^{(\prime)}},m_{\eta^{(\prime)}}), \qquad (68)\\
F^{\eta^{(\prime)}}_{BPV1} &= F^{\eta^{(\prime)}}_{tw2} \qquad (69)
\end{aligned}
$$

and $k_{\eta^{(\prime)}} = m_B^2/m_{\eta^{(\prime)}}^2$. The functions $Y_1(z,k,m)$ and $Y_2(z,k,m)$ are given by

$$\begin{aligned}
Y_1(z,k,m) = \quad & \frac{1}{(k-1)^2\, m^2}\Big[-2\,k\;(1-z) - k^2\,(1-z)^2 + 2\;(1-k\,z) \\
& +3\,k\;(1-z)\;(1-k\,z) - 2\,(1-k\,z)^2 - 2\;\ln(1-k\;(1-z)) \\
& +3\,k\;(1-z)\;\ln(1-k\;(1-z)) - k^3\,(1-z)^3\;\ln(1-k\;(1-z)) \\
& +k^3\,(1-z)^3\;\ln(-k\;(1-z)) + 2\;\ln(k\,z) - 3\,k\;(1-z)\;\ln(k\,z) \\
& +3\,k\;(1-z)\;(1-k\,z)^2\;\ln(k\,z) - 2\,(1-k\,z)^3\;\ln(k\,z) \\
& -3\,k\;(1-z)\;(1-k\,z)^2\;\ln(-1+k\,z) + 2\,(1-k\,z)^3\;\ln(-1+k\,z)\Big], \qquad (70)
\end{aligned}$$

$$\begin{aligned}
Y_2(z,k,m) = \quad & \frac{1}{(k-1)\; m^2}\Big[3\Big(-1+k\;(1-z)+k\,z \\
& -\ln(1-k\;(1-z)) - 2\;(1-z)\;\ln(1-k\;(1-z)) \\
& +2\,k\;(1-z)\;\ln(1-k\;(1-z)) + 2\,k\,(1-z)^2\;\ln(1-k\;(1-z)) \\
& -k^2\,(1-z)^2\;\ln(1-k\;(1-z)) - 2\,k\,(1-z)^2\;\ln(-k\;(1-z)) \\
& +k^2\,(1-z)^2\;\ln(-k\;(1-z)) + 2\;(1-z)\;\ln(1-k\;(1-z)-z) \\
& -2\,k\;(1-z)\;\ln(1-k\;(1-z)-z) \\
& -2\,(1-z)^2\;\ln(-k\;(1-z))\;\ln(\frac{1-k\;(1-z)-z}{1-z}) \\
& +2\,k\,(1-z)^2\;\ln(-k\;(1-z))\;\ln(\frac{1-k\;(1-z)-z}{1-z}) \\
& +2\,(1-z)^2\;\ln(1-k\;(1-z))\;\ln(\frac{-\,(1-k\;(1-z)-z)}{z}) \\
& -2\,k\,(1-z)^2\;\ln(1-k\;(1-z))\;\ln(\frac{-\,(1-k\;(1-z)-z)}{z}) + \ln(k\,z) \\
& +2\;(1-z)\;\ln(k\,z) - 2\,k\;(1-z)\;\ln(k\,z) - 2\;(1-z)\;(1-k\,z)\;\ln(k\,z) \\
& +2\,k\;(1-z)\;(1-k\,z)\;\ln(k\,z) - (1-k\,z)^2\;\ln(k\,z) \\
& +2\;(1-z)\;(1-k\,z)\;\ln(-1+k\,z) - 2\,k\;(1-z)\;(1-k\,z)\;\ln(-1+k\,z) \\
& +(1-k\,z)^2\;\ln(-1+k\,z) - 2\;(1-z)\;\ln(-z+k\,z) \\
& +2\,k\;(1-z)\;\ln(-z+k\,z) + 2\,(1-z)^2\;\ln(-1+k\,z)\;\ln(\frac{-z+k\,z}{1-z}) \\
& -2\,k\,(1-z)^2\;\ln(-1+k\,z)\;\ln(\frac{-z+k\,z}{1-z}) - 2\,(1-z)^2\;\ln(k\,z)\;\ln(\frac{-\,(-z+k\,z)}{z}) \\
& +2\,k\,(1-z)^2\;\ln(k\,z)\;\ln(\frac{-\,(-z+k\,z)}{z}) - 4\,(1-z)^2\,Li_2(k)
\end{aligned}$$

$$
\begin{aligned}
&+4\,k\,(1-z)^2\,Li_2(k)+2\,(1-z)^2\,Li_2\left(\frac{-\,(-1+k\;(1-z))}{z})\right)\\
&-2\,k\,(1-z)^2\,Li_2(\frac{-\,(-1+k\;(1-z))}{z})+2\,(1-z)^2\,Li_2\left(\frac{1-k\,z}{1-z}\right)\\
&-2\,k\,(1-z)^2\,Li_2\left(\frac{1-k\,z}{1-z}\right)\bigg)\bigg]. \qquad (71)
\end{aligned}
$$

Where $Li_2(x)$ is the dilogarithm or Spence function.

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Figure captions

FIG.1 Order α_s non-factorizable contributions in $B \to M_1 M_2$ decays.

FIG.2 The triangle Feynman diagrams for the transition form-factors of $g^* g^* - \eta'$ and $g^* g^* - \eta$.

FIG.3 The Feynman diagrams of the spectator hard scattering mechanism for B decays to $\eta' M$, where M is a light pseudoscalar or vector meson.

FIG.4 Branching ratios for B decays are shown as curves as a function of γ in units of 10^{-6}. The dashing curves are the results of conventional mechanisms, the solid curves are our estimations with incorporating SHSM contributions. The branching ratios measured by CLEO Collaboration are shown by horizontal solid lines. The thicker solid horizontal lines are its center values, thin horizonal lines are its error bars.

FIG.5 Direct CP violations in B decays involving $\eta^{(\prime)}$. Dashing curves are the predictions of conventional mechanism estimated with the QCD improved factorization approach. Solid curves are results when SHSM contributions added.

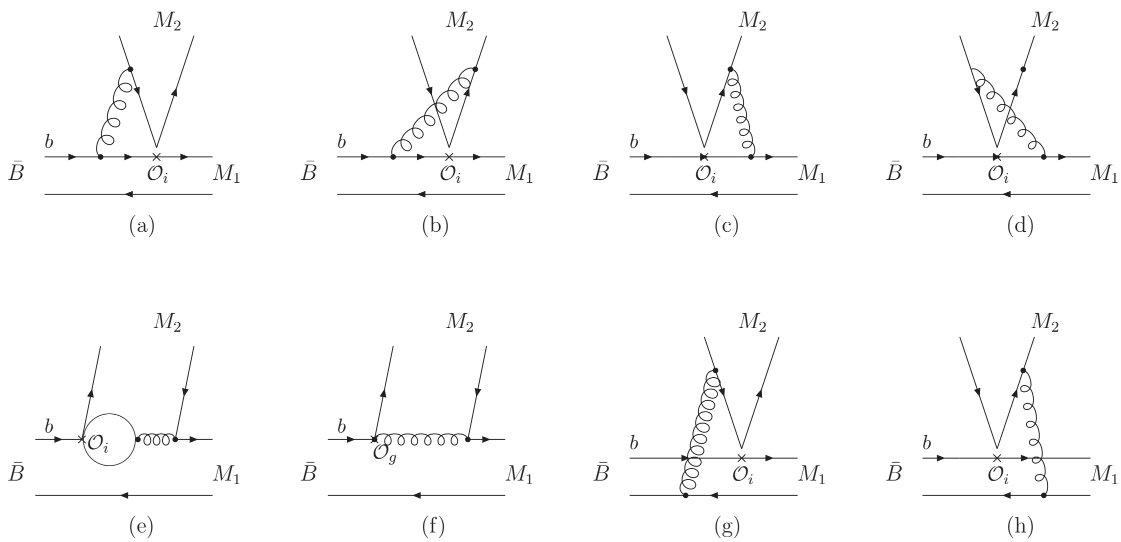

Figure 1:

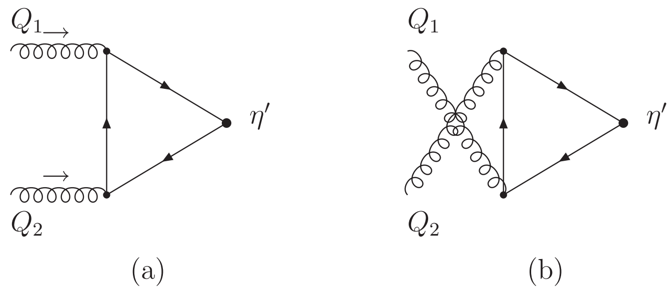

Figure 2

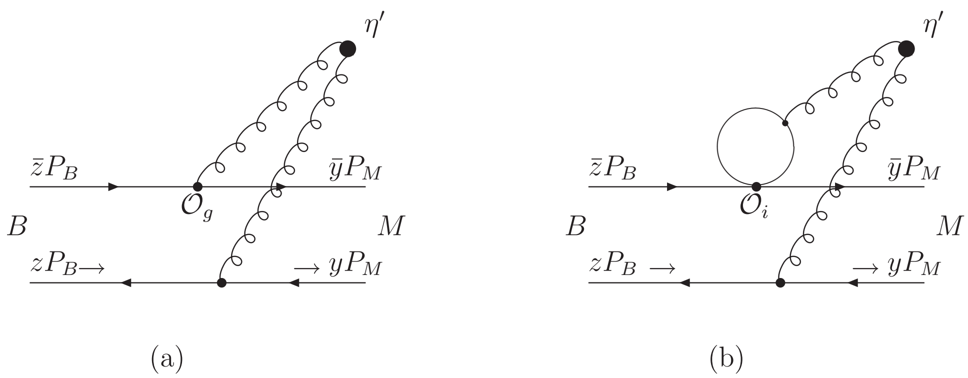

Figure 3

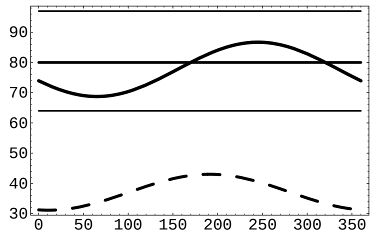

Fig.4.1, $Br(B^+ \to \eta' K^+)\, vs.\, \gamma$

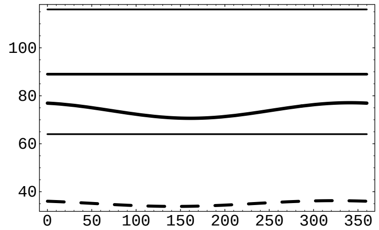

Fig.4.2, $Br(B^0 \to \eta' K^0)\, vs.\, \gamma$

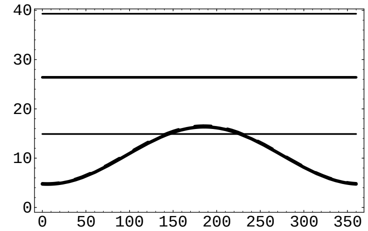

Fig.4.3, $Br(B^+ \to \eta K^{*+})\, vs.\, \gamma$

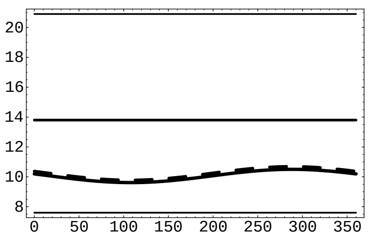

Fig.4.4, $Br(B^0 \to \eta K^{*0})\, vs.\, \gamma$

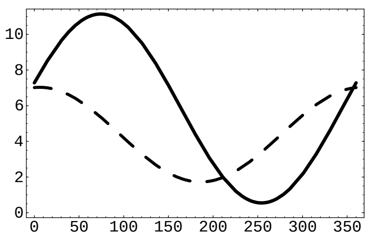

Fig.4.5, $Br(B^+ \to \eta' \pi^+)\, vs.\, \gamma$

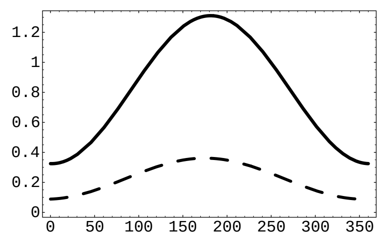

Fig.4.6, $Br(B^0 \to \eta' \pi^0)\, vs.\, \gamma$

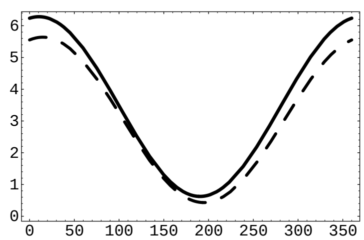

Fig.4.7, $Br(B^+ \to \eta' K^{*+})\, vs.\, \gamma$

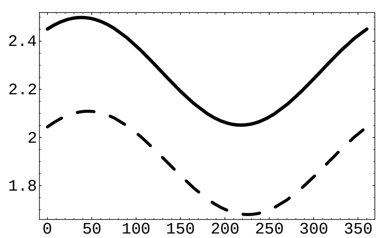

Fig.4.8, $Br(B^0 \to \eta' K^{*0})\, vs.\, \gamma$

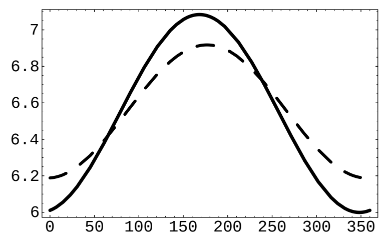

Fig.4.9, $Br(B^+ \to \eta' \rho^+)\, vs.\, \gamma$

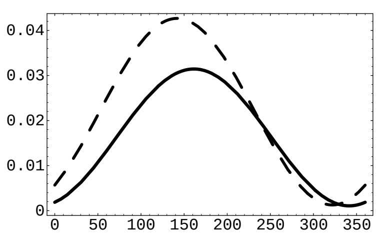

Fig.4.10, $Br(B^0 \to \eta' \rho^0)\, vs.\, \gamma$

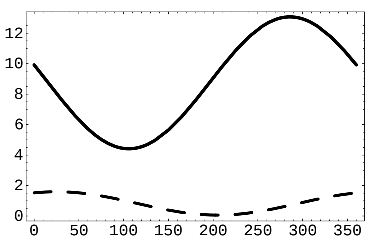

Fig.4.11, $Br(B^+ \to \eta K^+)\, vs.\, \gamma$

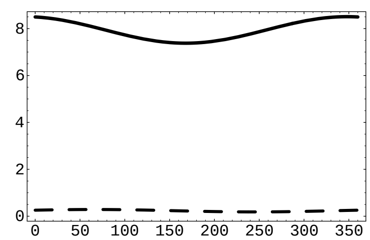

Fig.4.12, $Br(B^0 \to \eta K^0)\, vs.\, \gamma$

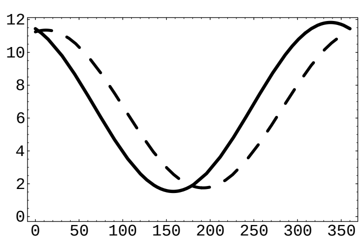

Fig.4.13, $Br(B^+ \to \eta \pi^+)\, vs.\, \gamma$

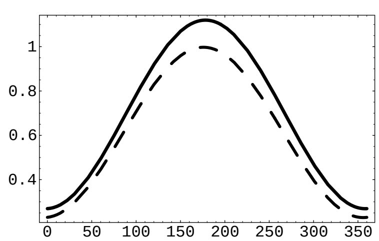

Fig.4.14, $Br(B^0 \to \eta \pi^0)\, vs.\, \gamma$

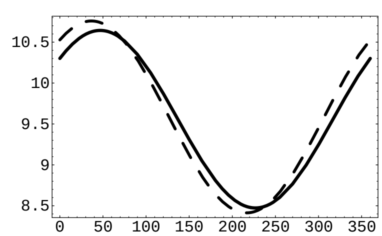

Fig.4.15, $Br(B^+ \to \eta \rho^+)\, vs.\, \gamma$

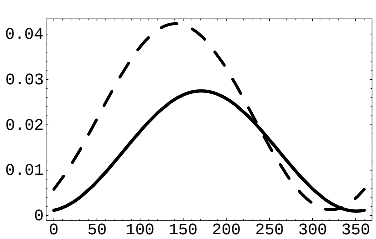

Fig.4.16, $Br(B^0 \to \eta \rho^0)\, vs.\, \gamma$

FIG 4

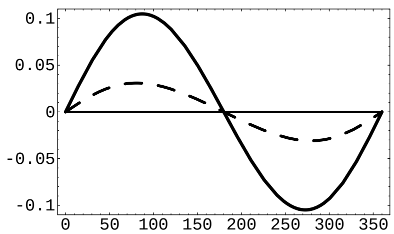

Fig.5.1, $\mathcal{A}_{cp}^{dir}(B^+ \to \eta' K^+)$

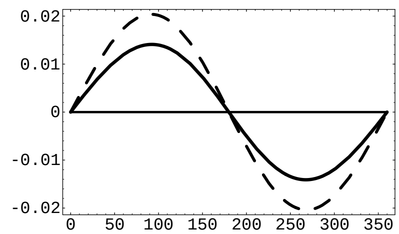

Fig.5.2, $\mathcal{A}_{cp}^{dir}(B^0 \to \eta' K^0)$

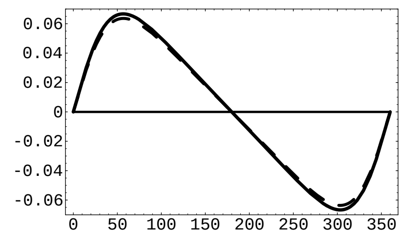

Fig.5.3, $\mathcal{A}_{cp}^{dir}(B^+ \to \eta K^{*+})$

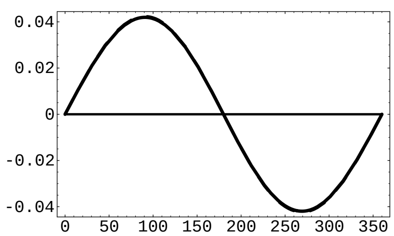

Fig.5.4, $\mathcal{A}_{cp}^{dir}(B^0 \to \eta K^{*0})$

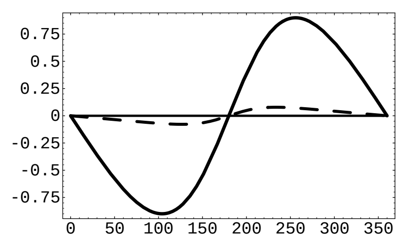

Fig.5.5, $\mathcal{A}_{cp}^{dir}(B^+ \to \eta' \pi^+)$

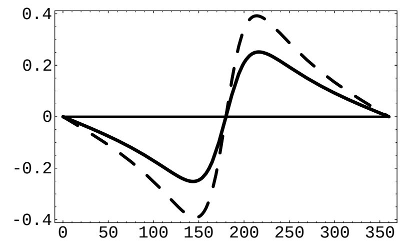

Fig.5.6, $\mathcal{A}_{cp}^{dir}(B^+ \to \eta' K^{*+})$

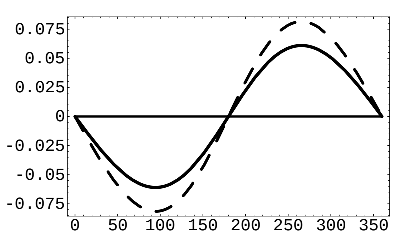

Fig.5.7, $\mathcal{A}_{cp}^{dir}(B^0 \to \eta' K^{*0})$

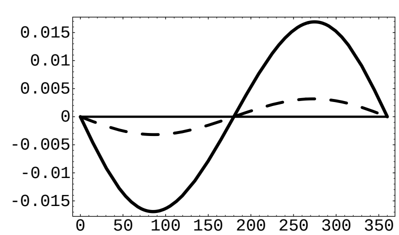

Fig.5.8, $\mathcal{A}_{cp}^{dir}(B^+ \to \eta' \rho^+)$

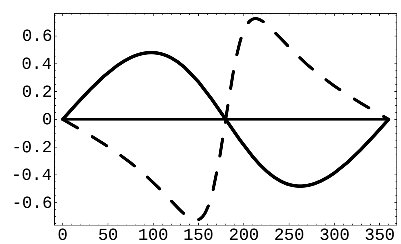

Fig.5.9, $\mathcal{A}_{cp}^{dir}(B^+ \to \eta K^+)$

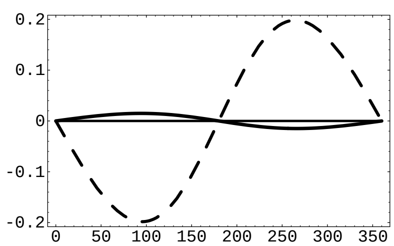

Fig.5.10, $\mathcal{A}_{cp}^{dir}(B^0 \to \eta K^0)$

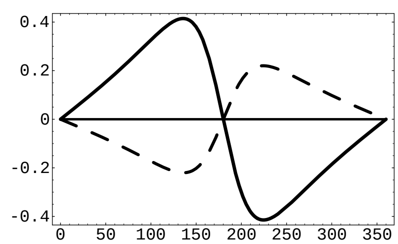

Fig.5.11, $\mathcal{A}_{cp}^{dir}(B^+ \to \eta \pi^+)$

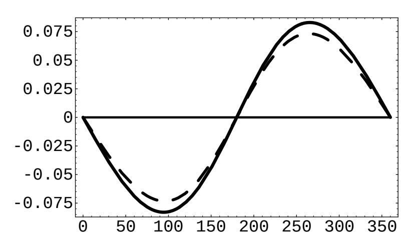

Fig.5.12, $\mathcal{A}_{cp}^{dir}(B^+ \to \eta \rho^+)$

FIG 5.